Environmental policies in competitive electricity markets

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CHAPTER 1

INTRODUCTION

1.1 General Introduction and Scope of the Thesis

The reduction of greenhouse gas pollution in electricity markets has been a political issue for several decades. Governments, businesses, and researchers are seeking for clean renewable alternatives for the traditional coal and gas plants. Consequently, in the last ten years there has been an uprising investment in renewable resources like wind and solar power. These investments contribute to meeting long term international commitments such as the Kyoto Protocol on the reduction of carbon dioxide (CO_2) emissions. We are not there yet, however. Since profit maximizing firms are inclined to invest in relatively cheap conventional technologies, regulators may have to put financial incentives as to make investments in clean but more expensive alternatives attractive. This dissertation considers several of these incentives. We aim to present a message for regulators and policy makers indicating the potential benefits and side-effects of policies. In addition, we provide a benchmark for how to achieve a desired market outcome in terms of resource allocation, pollution reduction, and other measures, by setting the right parameters in the available environmental policies. We state theoretical results based on analysis of a mathematical framework of the electricity market and numerical results that we obtain through the use and expansion of numerical tools.

In the remainder of this section we briefly explain how electricity markets work and give an overview of the different environmental policies. In Section

1.2 we elaborate on the methods and solution techniques used throughout the thesis. Section 1.3, which contains an overview of the main findings and contributions in each chapter, concludes the introduction.

1.1.1 Electricity Markets

In a typical (decentralized) electricity market, a couple of profit maximizing firms compete at several nodes in a network. These firms have two key decisions to make. First, firms have to decide for each node, in which of the available resources for power generation to invest and in how much capacity (in megawatt (MW)) of these resources to invest. There are many available resources, basically split into two categories. There are non-renewable resources or technologies such as coal, and certain types of gas. These technologies are also referred to as polluting, since with the production of power a certain amount of CO_2 is emitted. The second category consists of renewable resources or technologies such as wind power (onshore and offshore), solar power, biomass, and landfill gas. Such technologies are referred to as clean due to the low or negligible pollution that comes with their electricity production (besides other potential environmental damage caused during the life cycle of a power generator, see Weisser (2007)). Firms investing in these non-renewable and renewable technologies typically maximize their long term profits consisting of the total or expected profits from producing and selling power minus their investment cost. Investment is a long term decision.

A second decision firms make in each node is with which technologies to produce electricity and how much electricity to produce with each available technology. These decisions are made every short period, which could be a day, hour, or minute, and are short term decisions. A firm's objective in the short term is to maximize its profits from producing electricity. Production decisions may depend on demand for electricity, installed capacities, available capacities, the price of electricity, and several other factors.

The produced electricity is transmitted through the electricity network to consumers located at demand nodes. Such transmission is what distinguishes electricity markets and electricity networks from markets with other goods or services and transportation networks. Firstly, since power is a non-storable good, electricity generation at any time will have to go to a demand node and whenever there is demand, sufficient production capacity will have to be available. Secondly, power transmission from one node to another node may affect transmission capacities in the entire network in accordance with Kirchhoff's voltage law (see Chao et al. (2000)). Hence, when transmitting power between two nodes, the entire network and its capacities will have to be taken into account. This is done by a transmission system operator (TSO). A TSO decides on the power transmission or so-called flows between all the nodes in order to maximize their own short term profits from buying power at supply nodes and selling it to consumers at demand nodes.

Electricity demand is characterized by its low elasticity in the short term, since on a daily, hourly, or minutely basis consumers barely respond to changing electricity prices. Short term demand is however dependent on several other factors such as daylight and weather conditions. Therefore, electricity demand is typically fluctuating and subject to uncertainty.

Electricity prices in each node are commonly determined by the market, that is, prices will be such that the market is cleared. Depending on the number of firms and how the market is regulated, firms and the TSO either take these prices as given, or one or several decision makers are aware that by behaving strategically they can influence electricity prices. The former is the case when firms behave in a perfectly competitive environment, whereas the latter is the case in a monopolistic or oligopolistic setting. While there are often only a handful of electricity producing firms per region or country, which could potentially lead to firms exerting market power, perfect competition in electricity markets is generally strived for by regulators. They aim to mitigate any market power that firms may exert (Gilbert et al. (2004) and Helman (2006)).

In addition to firms, a transmission operator, and consumers, an environmental regulator is often involved in the market. An environmental regulator aims to reduce the burden of electricity production and consumption on the environment. Several policies are available to achieve this, as we discuss in the next section.

1.1.2 Environmental Policies

Firms are profit maximizing entities and are therefore inclined to invest in the cheapest available resources. Typically, the cheapest resources are also the most polluting ones (Sims et al. (2003) and Lise et al. (2006)). Governments or environmental regulators can interfere in the electricity market in order to provide financial incentives to make investments in cleaner resources more attractive. These incentives can be given for a number of political and economical reasons. Two main political arguments in favor of clean resources are the following. Firstly, there can be a target on emission reduction agreed upon by a group of countries or regulators, such as the climate policy of the European Union (EU) that concerns a binding greenhouse gas emission reduction target of 20% by 2020 (EU (2009a)). Secondly, there can be a target agreed upon by a group of countries or regulators on the amount or percentage of energy consumption that must come from renewable resources, such as in the EU Renewables Directive (EU (2009b)). While this thesis specifically focuses on these political targets, underlying these targets there are a number of economical reasons for giving financial incentives to cleaner (renewable) technologies. More investment in renewable technologies can result in enhanced energy security, technological advance, trade advantages, and skilled employment (Fischer and Preonas (2010) and EU (2009b)), as well as correct for potential positive externalities from learning spill-overs and learning by doing (Fischer and Newell (2008), Reichenbach and Requate (2012), and Kalkuhl et al. (2012)).

A distinction is made between policies that charge firms for their CO_2 emissions (and possibly other forms of pollution or damage to the environment) and policies that reward firms for investment in or production with renewable technologies in order to compensate for high investment costs in these cleaner alternatives. Another distinction can be made between quantity based and price based policies. A quantity based policy imposes a certain target or quota on the market. Firms can be rewarded for contributing to the target, punished for violating the target, or be obliged to buy a certain amount of permits in order to show compliance to the target. The value or price of a reward, fine, or permit is generally determined by the market. A price based policy, on the other hand, fixes a certain price that is rewarded to firms or charged to firms depending on the policy.

Since in Chapter 2 we deal with two policies that charge firms for their pollution, namely a cap-and-trade policy and a fixed taxation policy, we first discuss these briefly. Under a cap-and-trade policy, which is a quantity based policy, a regulator imposes a maximum CO_2 emission allowance level (referred to as the cap) on the market. For each unit of CO_2 emitted, a firm needs to buy a permit at a price that is determined in a secondary trading market. This price generally depends on the maximum allowed level and the demand for permits. When insufficient permits are obtained at the end of a certain period, typically a year, firms face a fine. The most notable cap-and-trade policy applied in practice is the European Union emissions trading system (EU ETS), which involves 31 European countries. Several states in the US also have cap-and-trade policies, for example the Regional Greenhouse Gas Initiative (RGGI), which is in effect in nine northeastern states, and the Western Climate Initiative (WCI), which is in effect in seven western states as well as four Canadian provinces.

In the fixed taxation policy, which is a price based policy, an environmental regulator is either charging firms with a fixed tax per unit of CO_2 emitted or charging consumers with a fixed tax per unit of CO_2 consumed. The so-called carbon tax is fixed for a certain period, typically a year or several years. Carbon taxation is effective in, amongst others, a few US states, several European countries such as Denmark, Norway, Slovenia, and Switzerland, and a few Asian countries such as India and Japan. A regulator can impose both an emission cap and a taxation, but firms are in some cases exempted from taxation if they participate in the emissions trading (for example in Switzerland). A policy combining fixed taxation and cap-and-trade, the so-called safety valve, has been suggested in the literature (Pizer (2001), Jacoby and Ellerman (2004), and Burtraw et al. (2010)). In a safety valve policy, as long as sufficient allowances are available the market acts like a cap-and-trade system; however, when the cap is reached, additional allowances can be bought at a fixed price. Such a policy can resolve the issues of a very high allowance price in case of a tight cap. Safety valves are out of the scope of this paper, but extending our mathematical framework with this policy is suggested for future research in relation to Chapter 2 of this thesis.

Several advantages and disadvantages of both policies are discussed in the literature. In favor of taxation, Avi-Yonah and Uhlmann (2009) state that it is easier to implement and hence can give an immediate price signal. Furthermore, taxation is argued to be more efficient than cap-and-trade in Aldy et al. (2008). Parry and Pizer (2007) mentions other possible advantages of carbon tax, such as carbon price certainty, more flexibility in changing the cost of carbon over time, and direct income for the regulator. In favor of cap-and-trade however, Fischer and Springborn (2011) find that the market is less sensitive to productivity shocks under cap-and-trade. Additionally, in general there seems to be more political resistance to taxation than to cap-and-trade, and cap-and-trade allows regulators to put a tighter cap every year such that emissions are progressively curbed (Parry and Pizer (2007)). We stay away from a direct comparison of the two policies and instead present a thorough theoretical and numerical analysis of both policies, which will be further discussed in Section 1.3.

As opposed to making polluting technologies more expensive, a regulator may also make clean renewable technologies more financially attractive by handing out (indirect) subsidies. We explain two of these policies, namely a renewable energy obligation, which is dealt with in Chapter 3 and feed-in tariffs, which is the policy addressed in Chapter 4. Under an obligation policy, a regulator imposes a renewable energy obligation on the electricity market. The obligation is a quantity based policy instrument prescribing a minimum quantity of power that should be produced with renewable energy resources. It is often expressed in terms of a fraction of the total production. In order to show compliance to the obligation, firms need to present a sufficient number of so-called green energy certificates at the end of each period, typically several months or a year. In principle each unit production with a renewable resource is rewarded with one certificate, but in the UK electricity market different technologies are eligible for a different number of certificates, so-called technology banding. In the latter policy, the obligation shifts from one on renewable production to one on certificates. Technology banding is done mainly to support less established renewable technologies. Green certificates can be traded on a secondary market and will thus have a value depending on the demand for certificates. The value of a certificate represents a reward for production with a renewable resource and adds to the short term profits of firms. When not committing to the target, firms are charged a fine. Renewable obligations are effective in a few European countries such as Belgium, Italy, and the UK, and in 29 US states where the so called Renewable Portfolio Standard (RPS) is in effect.

Under a feed-in tariff (FIT) policy, a regulator rewards a unit production with renewable resources with a price that is different from (and often higher than) the price of electricity. This direct form of subsidy can be different for different technologies and is paid either out of national taxes or by electricity consumers. The most common feed-in tariff policies are the fixed FIT policy and the premium price policy. A fixed FIT is a fixed payment per unit production with a renewable resource, paid to firms instead of the electricity price. The tariff is typically fixed for several years. In the premium price policy, per unit production with a renewable resource a fixed price is paid on top of the electricity price. While the tariffs are fixed for a certain period, typically several years, the day to day payment in such a scheme can be fluctuating. Fixed FIT policies are applied in most European countries and some US states. The premium price policy is used in for example Czech Republic, Slovenia, and Estonia. In the Netherlands, the so-called Dutch Spot Market Gap policy is applied. This policy is a combination between a fixed FIT, paid when the electricity price is below a certain level, and a variable premium price, paid when the difference between the fixed tariff and electricity price exceeds a certain boundary. In Spain a cap-and-floor system, which also combines features of both the fixed FIT and the premium price policy, is in effect.

A number of studies compare green certificate systems and feed-in tariffs. Green certificate system are considered more cost-effective in theory (Menanteau et al. (2003), Palmer and Burtraw (2005), Böhringer et al. (2007), and Fell and Linn (2013)). However, there are practical examples that show that obligation policies can be more costly to society than feed-in systems (Butler and Neuhoff (2008)) and less successful in terms of encouraging investments in renewable technologies (Mitchell et al. (2006)). Furthermore, obligation policies may single out the cheapest renewable technologies (Meyer (2003), Wood and Dow (2011), Verbruggen and Lauber (2012), and del Rio and Gual (2007)). Johnstone et al. (2010) empirically establish that while obligation policies manage to induce technological innovation, feed-in tariffs may be necessary for inducing innovation in more expensive technologies. Chapter 3 of this thesis suggests that the UK banding policy and an alternative banding policy that we introduce can also accomplish this. It is worth noting that budget constraints may force governments to switch to less costly, but possibly less efficient, policy instruments (del Rio and Gual (2004)).

Finally, in the economics and policy literature several claims are made about policy interactions. When a regulator's policy goal is to reduce greenhouse gas emissions as opposed to meeting a renewable quota, subsidizing renewables is not cost-effective, neither instead of charging firms for emissions (Palmer and Burtraw (2005)), nor in combination with cap-and-trade (Böhringer and Rosendahl (2010)). In the latter case, supporting renewable technologies may even induce investments in polluting non-renewable technologies. It is also argued that imposing cap-and-trade and renewable quota can lead to substantial excess cost (Böhringer et al. (2009)) and that feed-in tariffs in combination with cap-and-trade can lead to higher CO_2 emissions in a duopoly market (Chaton and Guillerminet (2013)). In this thesis, we focus on each policy separately and do not consider policy interactions.

1.2 Methodology

Throughout the thesis, we work with a mathematical framework to represent the electricity market. Using this mathematical framework, we analyze and if possible characterize the (equilibrium) outcome of the market for different policies and different parameter choices within these policies. Due to the complicated nature of the electricity market and the size of the mathematical problem, it is not always possible to analytically describe the possible market outcomes. In such cases, we have numerical tools at hand with which we can still track or approximate the market outcome. In this section, we briefly elaborate on our modeling choices and assumptions, and we explain the methods used throughout the thesis.

Since we deal with both long term and short term decision making, these two types of decisions are dealt with separately, in two stages. Long term investment decisions are done at a first stage, while short term production and power dispatching to consumers are done at a second stage. When multiple firms are competing at both stages, we are dealing with a two-stage game. This game can be modeled in a mathematical way.

Several two-stage models for electricity investments are available in the literature, which can basically be separated into two streams, one dealing with imperfect competition models (Murphy and Smeers (2005), Ralph and Smeers (2006), Yao et al. (2008), Ruiz et al. (2012), and Hu and Ralph (2007)) and the other dealing with perfect competition models (Neuhoff et al. (2005), Ehrenmann and Smeers (2008), Zhao et al. (2010), and Gürkan et al. (2013)). When a small number of firms compete, an imperfect oligopolistic framework is believed to be a better representation of reality. On the other hand, when market power is successfully mitigated by regulators, a perfect competition framework is better suited. In addition, from a modeling and analyzing perspective, perfect competition first and foremost presents a benchmark for imperfect competition models and leads to more analytically tractable results; that is, assuming imperfect competition in a two-stage framework could result in an equilibrium problem with equilibrium constraints (EPEC), which is known to often be analytically intractable due to its non-convexity (Gabriel et al. (2012)). We thus choose to focus on a perfectly competitive electricity market and will in each chapter of this thesis use and expand the two-stage model as presented by Gürkan et al. (2013). Their analysis specifically focusses on capacity investments and resource adequacy in both a deterministic and a stochastic setting, which poses a good starting point for incorporating and subsequently analyzing different types of environmental regulation.

In the model for the two-stage game, a number of firms simultaneously invests in production capacity in the available technologies at the first stage. Their objective is to maximize long term profits, consisting of (expected) profits from electricity production minus their investment cost. Electricity production takes place at the second stage, where firms produce a certain amount of power as to maximize their short term profits. In addition, the transmission system operator dispatches power between supply and demand nodes as to maximize its short term profits from buying power at supply nodes and selling it at demand nodes. The second stage is cleared by means of two market clearing conditions, one condition balancing electricity supply and demand while setting the electricity price, and one condition putting a price cap on the electricity price in case there is demand curtailment. Firms and the TSO are assumed to be price takers at both stages, since we are in a perfect competition setting. In the models in Chapters 2 and 3, all the second stage problems can be solved as a single optimization problem, a so-called optimal power flow (OPF) problem. This was originally explored by Boucher and Smeers (2001).

In summary, we deal with a two-stage problem with firms investing at the first stage and, if existent, an OPF problem at the second stage. All constraints, objective functions, and cost functions at both stages are linear, except in Chapter 4, where we also consider (non-linear) convex cost functions. The first and second stage problems are linked to each other in the following way. In each firm's objective function at the first stage, the optimal second stage production quantities for given investment quantities are taken into account. At the second stage, optimal production quantities are limited by first stage investment quantities via a capacity constraint. The dual variable to this capacity constraint, referred to as the scarcity rent, represents the additional second stage profit that comes with an additional available unit of capacity. Using this information, firms decide whether or not to invest. More specifically, a firm invests in a technology in a certain node if the (expected) scarcity of that technology in that node covers the corresponding unit investment cost. Second stage sensitivities are thus key in determining the optimal first stage decisions. When none of the firms nor the TSO have an incentive to deviate from a certain solution at neither stage, the market is at equilibrium.

In the deterministic version of the model, first and second stage decisions are basically made at the same time. While a stochastic representation may be more realistic, in the deterministic setting results remain analytically tractable in stylized versions of the model. Since investment and production decisions are made simultaneously, for all firms in all nodes and for each technology, production quantities are generally equal to available investment quantities. In addition, the optimal scarcity rent in each node for each technology equals the corresponding investment cost. These two observations, that are also made in Gürkan et al. (2013), allow us to write the two-stage game as a single linear optimization problem when all cost functions are linear. As such, we can apply standard linear optimization techniques to analyze the problem and characterize the equilibrium to the two stage game. Analysis of the deterministic problem is mainly done in Chapter 2 and briefly in Chapter 3.

Many short term (second stage) parameters are typically fluctuating and unknown at the time firms invest in production capacity. This uncertainty is taken into account in the stochastic version of the model. In particular, consumer demand and available capacity of renewable resources like wind and solar power are considered unknown to firms at the first stage and are revealed at the second stage. At the first stage, firms only know the distributions and expectations of the random variables involved, but not the realizations. Contrary to the deterministic model, we can no longer write the two-stage game as a single optimization problem. When the second stage problems can be written as a single optimization problem (the OPF), like in Chapters 2 and 3, then the two-stage game is in general equivalent to a standard two-stage stochastic program. However, instead of dealing with two-stage stochastic programs, we derive the Karush Kuhn Tucker (KKT) conditions of both stages of the problem and write the problem as a mixed complementarity problem (MCP). We do this for two main reasons. First of all, an MCP is a more suitable framework for adding regulator's conditions related to environmental policies on production and prices. In some cases the problem can no longer be written as a two-stage stochastic program, while the problem can still be written as an MCP. In particular, in the alternative banding policy introduced in Section 3.4.2, there is an additional condition that involves both primal and dual variables, and in the fixed feed-in system in Chapter 4 there is, to the best of our knowledge, no single optimization problem that solves the second stage. Secondly, as we explain in more detail below, we have optimization tools at our disposal that are capable of handling large sized MCPs. For these reasons, and in order to be consistent in our numerical studies, we formulate all models as MCPs.

Several previous studies analyze electricity and energy markets using complementarity problems, such as Hobbs (2001), Metzler et al. (2003), Bushnell (2003), Gabriel et al. (2005), Kazempour et al. (2011), Chen et al. (2011), Ehrenmann and Smeers (2011), and Shanbhag et al. (2011). In other areas of operations research like supply-chain management (Adida and DeMiguel (2011)) and transportation (Agdeppa et al. (2007)) complementarity problems are also used. For electricity markets there also exists literature on an even broader class of mathematical problems, for example the quasi-variational inequality (QVI) formulation in Hobbs and Pang (2007). The QVI results from piecewise linear demand functions and joint constraints, and is considered more challenging to analyze, both theoretically and numerically, compared to MCP formulations.

Throughout the thesis, the mixed complementarity problems are dealt with in two ways. First, in some situations, model outcomes can be expressed in terms of expectations and hence an equilibrium solution can be approximated analytically. Such approximations are used for determining parameters in a regulator's policy in a way that certain policy goals are met. In particular, we use this type of analysis in Chapter 4 to determine the feed-in tariffs in a way that at least a certain fraction of electricity production is done with renewable resources, as well as to obtain feed-in tariffs that, in expectation, lead to a desired mixture of renewable technologies.

Second, the MCP can be analyzed numerically, using a sampling technique. Given probability distributions for the random parameters in the model, we generate random samples using a pseudorandom number generator. Throughout the thesis we work with uniform distributions for all random variables and thus sample from uniform distributions. We obtain realization vectors, simply referred to as realizations. For each realization there is a second stage problem that needs to be solved. At the first stage, expectations are replaced by sample averages. Then we obtain a large sized MCP that we program in GAMS and solve using the PATH solver, see Ferris and Munson (2000). This solver, which can be seen as a generalization of Newton's method for solving MCPs, finds an equilibrium solution to the set of KKT optimality conditions of the sampled first and second stage problems. Since solving these large MCPs can be very time consuming for large instances of the problem, we apply sampling to relatively small networks. We use samples of size 3000, which we established to be sufficiently large by subsequently solving the model for 3, 100, 1000, 3000, 5000, 8000, and 10000 samples. In all models, there is a significant difference between the solutions for 100 and 1000 samples. However, increasing the sample size to 3000 does not reveal significant changes, nor does increasing the sample size any further. We solve the MCP for a range of parameter values. Per instance we solve, computation times are around 15 minutes (fixed taxation) and around three hours (cap-and-trade) in Chapter 2, around two hours in 3, and around 20 minutes in 4, on a 300MHz Pentium-II with 1 GB RAM.

1.3 Contributions and Outline of the Thesis

The rest of this thesis consists of three chapters. In each chapter we focus on a different environmental policy. These policies are analyzed in a mathematical model of the electricity market. While the model used in each chapter is similar, it is introduced in its entirety in each chapter and consecutively extended to include the necessary modifications for the policy we consider.

In Chapter 2, which is based on Gürkan, Langestraat, and Özdemir (2013), we analyze and compare the effects of cap-and-trade and fixed taxation on investments, production, pollution, and consumer prices. There currently exists a broad literature on the effects of taxation and in particular on the impact of cap-and-trade policies, more specifically of the EU ETS. Chen et al. (2008) and Lise et al. (2010) use the COMPETES (Comprehensive Market Power in Electricity Transmission and Energy Simulator) model to investigate short-term effects of the EU ETS, in particular with respect to cost pass through to consumers and what factors affect this cost past through. Our modeling framework is more stylized and we theoretically confirm their result that under perfect competition there is a cost pass through of cap-and-trade of 100%. There are also studies specifically focussing on the impacts of the EU ETS on the Spanish electricity market (Linares et al. (2010)) and the Italian electricity market (Bonenti et al. (2013)). We instead study a more general framework that is representative for any electricity market. Lise and Kruseman (2008) present a recursive dynamic model of the electricity market used for simulations and focus on long-run implications of cap-and-trade. They conclude that perfect competition can beneficial for the environment, since under perfect consumption lower emissions are observed. The models contained in the literature discussed so far are more extensive than our model and are mostly used for simulation studies. The model in Ehrenmann and Smeers (2008) is more similar to our model, but their focus is specifically on the effects of different price caps under the assumption of random fuel prices, as opposed to random demand that is assumed in our work.

The first part of our analysis in Chapter 2 will be strictly theoretical and based on a very stylized mathematical representation of the electricity market, with no network effects and deterministic demand. Due to the growing amount of research in this field over the last decade, some of our findings (partly) overlap with known results in the literature. Nonetheless, we provide theoretical confirmations of previous numerical findings and establish a few new results relevant to policy makers. Our key results are the following:

- In case of cap-and-trade, either one or a mixture of two technologies is used at a market equilibrium. Such a mixture consists of a relatively clean and a relatively dirty technology. In the absence of a ceiling on total emissions, marginal operating costs of different technologies form a fixed merit order; that is, the marginal costs are ordered in an ascending fashion. Based on the observed demand, this merit order is used to determine the total number of technologies used so that all demand is satisfied. We show that, as long as there is enough capacity in the system, when a fixed maximum allowance level is introduced, different demand levels impose different prices for a unit of emission allowance, and consequently there is no fixed merit order on the technologies. Therefore, for different levels of observed demand one can find a different optimal mixture. This confirms numerical results on merit order changes in for example Zhao et al. (2010).

- For the cap-and-trade model we develop an algorithm for finding the induced optimal technology mixture in a systematic way. Using this, we show that the price of electricity and the price of allowances increase as the maximum allowance level decreases.

- In contrast to cap-and-trade, when a fixed tax is charged for the emissions, the merit order is fixed for all demand levels. The first technology in the merit order, if unique, is the only generating unit. Therefore, a cap-and-trade system often results in a wider portfolio of technologies than fixed taxation.

- For a special case of taxation, that is, when the fixed tax is equal to the optimal price of emission allowances for some given maximum allowance level, we find a range of equilibria. Some of these equilibria satisfy the emission allowance level, while others do not.

- Given the cost characteristics and the amount of CO_2 emission per unit production for each technology, one can analytically identify which technologies will never be in the optimal technology mixture under either policy. Such technologies are unattractive for investors, and are determined by our analysis.

In the second part of Chapter 2 we consider the stochastic version of the model and carry out a numerical study. A small network with three producing firms and three technologies is considered. We investigate the effects of stochastic demand and effectivity of cap-and-trade and taxation when there is limited transmission capacity in the network. Our key findings are the following:

- Uncertainty leads to broader optimal mixtures of technologies. At demand peaks, peak load technologies that are characterized by low investment cost and high production costs enter the optimal production mixture.

- If there is a shortage of transmission capacity in the system, only introducing financial incentives and instruments (taxation or a cap-and-trade system) neither is sufficient to curb CO_2 levels nor necessarily induces investment in cleaner technologies. As such, investments in transmission capacity may be necessary in order to achieve the desired pollution reduction.

- Similar to the deterministic case, we consider the effects of setting the fixed taxation level equal to an optimal price of emission allowances found for some maximum allowance level. Then either we find the same solution, or there are multiple optimal solutions. In the latter case, there is a trade-off between minimizing pollution and maximizing the regulator's profits or surplus. Again, some of the solutions violate the emission allowance level, meaning that a taxation chosen in this way cannot achieve the same as a cap-and-trade system without explicitly enforcing a cap on firms.

In Chapter 3, which is based on Gürkan and Langestraat (2013), we deal with renewable energy obligations and green certificates. We model the renewable obligation into the two-stage mathematical framework and introduce the UK technology banding. To the best of our knowledge, there are no models dealing with technology banding available in the literature. One theoretical concern of the UK banding policy is that it may result in an outcome where the original obligation target on renewable production is not satisfied, hence potentially resulting in more pollution. We therefore present an alternative banding policy, in which the obligation target is still on electricity production. We provide revenue adequate pricing schemes for the three obligation policies above in order to guarantee that the regulator can cover the payments to firms owning certificates with the mark-ups paid by consumers. We carry out a numerical study using the stochastic version of the model with uncertainty in both consumer demand and available capacities of renewable resources. We investigate the potential effects and side-effects of a renewable obligation, the

UK banding policy, and the alternative banding policy, and find the following key results:

- An obligation can reduce pollution in two ways. Firstly by the replacement of polluting non-renewable technologies by cleaner renewable technologies. Secondly, due to the random availability of renewable resources, the obligation may lead to replacement of polluting non-renewable technologies by (slightly) cleaner non-renewable technologies.
- Although technology bandings are introduced to give financial incentives to less established technologies, when the obligation target is too low, bandings are not necessarily effective in giving these incentives.
- As expected, the UK banding policy cannot guarantee that the original obligation target is met. In particular, when there is a lot of investment in a technology with a high banding coefficient (namely offshore wind), the new target on certificates is met while the original target on production is not.
- The alternative banding policy provides a way to make sure that the original target is met while supporting less established technologies, but it comes with a significantly higher consumer price.
- As a concerning side-effect of technology banding, we find that a cost reduction in a technology with a high banding leads to more CO_2 emissions under the UK banding policy and to significantly higher consumer prices under the alternative banding policy. When investment costs in technologies reduce over time, banding coefficients should be adjusted accordingly.

In Chapter 4, which is based on Langestraat (2013), we take a closer look at feed-in tariff systems. In particular, we deal with a fixed FIT policy and investigate whether or not quantity based policy goals can be achieved by means of this price based policy instrument. We consider the electricity market investment model with uncertainty in both demand and availability of renewable resources and provide a number of analytical results for different assumptions on the investment cost functions of renewable technologies. More specifically, we distinguish between renewable technologies having linear investment cost and renewable technologies having non-linear convex investment cost and establish significantly different results for the two cases. We assume the regulator to have the freedom to choose feed-in parameters, specifying for each technology the relative feed-in tariff they receive. This way, a regulator differentiates between technologies as to achieve broader technology mixtures or to give more financial support to less established technologies. While non-linear convex cost are sometimes assumed in the literature on electricity markets (Reichenbach and Requate (2012) and Traber and Kemfert (2011)), to the best of our knowledge there is no in-depth theoretical analysis of the consequences of feed-in policies under the different cost assumptions, in particular in the presence of uncertainty. Our main analytical findings are the following:

- A renewable energy obligation is satisfied at the market equilibrium, regardless of assumptions on investment costs. A lower bound for the optimal price of certificates is provided.
- Under the assumption of linear investment cost in renewable technologies, a fixed feed-in tariff cannot guarantee that a certain obligation target is met. Using a benchmark model that imposes an explicit obligation on the market, we find a feed-in tariff such that the obligation target can potentially be achieved. However, when the obtained feed-in tariff is handed out to firms while the explicit obligation is omitted, multiple solutions occur. Only one of these solutions meets the obligation and there is no guarantee that this solution is going to be the one achieved by the market.
- Under the assumption of non-linear convex investment cost in renewable technologies, a fixed feed-in tariff achieves the same as an obligation when the right feed-in tariff is chosen. We find the correct fixed feed-in tariff using the benchmark model with an explicit obligation on the market.
- In case of linear investment cost functions, a regulator can choose any subset of technologies that she prefers to support, and set the feed-in tariffs in a way that all other technologies are not in the optimal technology mixture. Since there exist multiple optima, it is not guaranteed that technologies in the chosen subset are actually in the optimal technology mixture.
- For the case of non-linear convex investment cost functions, we find for which choices of feed-in parameters each technology is singled out in the optimal mixture. Additionally, we derive in which way to change the feed-in parameters such that a second technology is added to the optimal mixture.

- When investment cost functions are quadratic and when there are three renewable technologies in the market, we analytically establish for any choice of feed-in parameters the corresponding optimal mixture. Hence, besides parameters for which one technology is singled out, we find the parameters for which any two or all three technologies are in the optimal mixture.

We finally carry out a numerical study with three renewable technologies and consider the effects of different feed-in parameter choices in case of linear and quadratic cost functions, as well as two other non-linear convex cost functions. These are our key contributions:

- For the case of non-linear non-quadratic convex cost functions, when analytically we are only able to find the parameter choices for which one technology is singled out, we found a way to numerically trace the optimal technology mixture corresponding to any choice of parameters. One can thus use our numerical tools for finding feed-in parameters resulting in any desired optimal mixture of technologies.

- Our numerical observations lead to a theoretical generalization for all cost functions with either a superadditive or a subadditive non-constant part of its marginal cost function. In particular this means that when a regulator faces a market with firms having investment cost functions of one of these types, our analysis as well as our graphics provide a useful guideline for determining feed-in tariffs in a way that guarantees that a certain target on renewable electricity production is met and resulting in any desired optimal technology mixture of up to three renewable technologies.

CHAPTER 2

Introducing CO_2 Allowances, Higher Prices for All Consumers; Higher Revenues for Whom?

2.1 Introduction

These days, policy makers and businesses are setting goals in order to reduce the emission of carbon dioxide (CO_2). Especially power generators in electricity markets emit high levels of CO_2. Since investments in polluting technologies such as coal are most profitable, profit-maximizing firms do not have the right incentives to invest in cleaner alternatives.

Encouraging firms to invest in cleaner technologies can be done by implementing financial incentives. We consider two actions governments may take to accomplish such incentives. One is imposing a maximum allowance level on the total emissions by power generators. Firms buy and trade permits on a secondary market, which results in a price for emission allowances that should be paid when allowances are scarce. This is what is called a cap-and-trade system. Another way to give firms financial incentives to invest in cleaner alternatives is to charge a fixed tax per unit emission. Our goal is to analyze the effects of introducing these incentives on investment and production quantities, and on consumer prices. Our analysis will be done in two parts, the first part focussing on a deterministic demand setting in order to get more stylized and analytical results, and the second part dealing with stochastic demand as to derive results that are closer to reality.

We are going to consider the electricity market in a perfect competition setting. For perfectly competitive electricity markets, emission allowances have been analyzed before in for example Zhao et al. (2010), Chen et al. (2008), and Lise et al. (2010). However, these papers put more emphasis on the initial allocation of allowances and the effects thereof. In addition, the models are more general and are therefore used for deriving mostly numerical results, while we consider a more stylized framework in order to derive analytical results. Finally, Ehrenmann and Smeers (2008) deal with a perfect competition model including CO_2 emission allowances and endogenous allowance prices. They study the effects of different price caps in case of demand curtailment and of uncertainty in fuel prices and environmental policies. While we adopt their way of modeling emission allowances, we put more emphasis on the effects of the policies themselves and rather than taking fuel prices and policies as uncertain, we are going to deal with uncertain demand.

The electricity market can be seen as a two-stage game between firms, where investments take place at the first stage, and production and dispatching to consumers take place at the second stage. As mentioned in Chapter 1, a suitable modeling framework is presented by Gürkan et al. (2013). We first deal with a deterministic version of this model and extend it to include cap-and-trade and taxation. We show that under either policy the two-stage game can be reduced to a single optimization problem, similar to what has been shown for the original problem. For a stylized version of the single optimization problem we then analyze equilibria under both cap-and-trade and taxation. We make two simplifying assumptions. First, we consider a single node and assume that each firm is producing with a single and unique technology. Having a single node means that we can ignore network limitations that may affect the tractability of results. Second, we assume an order on the technologies and their CO_2 emissions; that is, we can order the technologies from lowest to highest marginal cost and assume that the cheapest technology is the most polluting, the second cheapest is the second most polluting, and so on. This assumption is both realistic and, as we show, can be made without loss of generality. These assumptions allow us to systematically analyze the direct effects of cap-and-trade and taxation on investments. These effects on investments can best be explained via the notion of merit order. At the beginning of a period firms invest in certain technologies; this is the first stage. Then, at the second stage, firms use the installed capacities to generate power; a transmission system operator (TSO) will buy the power and dispatch it to the demand nodes. Power is dispatched to the demand nodes according to a merit order. A merit order is a sequence of technologies based on their marginal costs in an ascending fashion. When power is dispatched, power from the first technology in the merit order will be used until all its capacity is used up. Then, the next technology in the merit order will be used, and so on. That way a number of technologies will be used to satisfy demand. In each node, the market price is then set by the technology that is active on the market and producing with the highest marginal cost. Regulation via either a cap-and-trade system or a fixed tax affects marginal costs since it comes with an additional cost per unit production. Obviously, these marginal cost changes may affect the merit order and hence the dispatching order. Production quantities and hence investment decisions may change accordingly.

In case of cap-and-trade, firms will have to pay the unit allowance price per unit emission. This allowance price is determined by the market, and as such is dependent on both the level of demand and the maximum emission allowance level. Therefore, marginal costs and hence the merit order may change with a change in demand or allowance level. This analytical result coincides with numerical evidence found in Zhao et al. (2010), who carry out a numerical study and find that the merit order changes for certain (high) allowance prices. Since in our analysis we find the allowance price to be increasing when the allowance level goes down or when demand goes up, we have a non-fixed merit order. We show that in our simplified setting either one or two technologies are first in a merit order, for given demand and allowance level. Three different cases can be distinguished: First, the dirtiest and cheapest technology can satisfy the demand without violating the (relatively low) allowance level. This technology will be the first in the merit order and hence the only technology used at the market equilibrium. Total emissions will be below the allowance level resulting in the allowance price to be zero (free allowances). Second, even the cleanest firm cannot satisfy the demand while meeting the (very strict) allowance level. The most expensive technology will be first in the merit order and hence the only technology used. Electricity demand will not be satisfied and the electricity price will be set at a price cap. Third, when none of these two cases occur, a combination of a relatively cheap and dirty and a relatively expensive and clean technology will be first in the merit order. The allowance price is then set in such a way that marginal costs of these two technologies are equal. We develop an algorithm to find the optimal technology mixture and the resulting allowance price in a systematic way. Using the algorithm, we show that electricity prices and the price for emission allowances increase when the emission allowance level decreases.

In case of a fixed tax, the merit order is fixed in the sense that it does not depend on the demand or the allowance level. Either one or two technologies are first in the merit order, which can easily be found by comparing effective marginal costs. A special case is when the taxation is set equal to the optimal unit allowance price that was found in case of cap-and-trade for a given allowance level. We find that multiple market equilibria exist, of which some do not satisfy the allowance level. Finally, we characterize technologies that will never be first in the merit order; that is, technologies for which there is no level of taxation such that it becomes the cheapest. We show that this, in turn, implies that those technologies will never be in the optimal mixture in case of cap-and-trade either.

After analyzing the effects on firms, we take a look at the extent to which costs for CO_2 emissions are passed through to consumers. Several results for perfect competition with inelastic demand have been shown in the literature; see for example Bonacina and Gullí (2007) and Chen et al. (2008), where it is argued that there is a 100% cost pass-through to consumers. Contrary to our study, they take the price of emission allowances as exogenous to the model and in addition focus on the short-run without considering investment strategies. Even though an endogenously determined allowance price results in a different merit order for different demand levels, we show that both endogenous prices and the two-stage nature of the model have no effect on the cost pass-through rate as long as demand is not curtailed; that is, the cost pass-through is still 100%. If demand is curtailed we show that the pass-through even exceeds 100%.

The second part of our analysis deals with the two-stage investment model with stochastic exogenous demand. Demand is unknown to firms at the first stage and will be revealed at the second stage. We can now interpret the first stage as the long-run; that is, investment decisions are made once every period, for example a year, while not knowing future demands. The second stage can be seen as the short-run, where each day there is a demand realization. Each day a second stage problem is solved, while investment decisions are made based on the expected outcome of these daily realizations. To the model as presented by Gürkan et al. (2013) we again add the cap-and-trade and fixed taxation. As allowances are typically set for a certain period rather than on a daily basis, the maximum emission allowance is going to be imposed at the first stage. As stochastic programs are obscured by the large dimension of the problem, instead of an analytical study we carry out a numerical study using sampling. We propose the sampled versions of the problem in the form of a large mixed complementarity problem (MCP). Using the PATH MCP-solver we can derive and analyze numerical results. In particular, we focus on the adequacy of cap-and-trade and fixed taxation in the presence of limited network capacity.

For our numerical study we consider a small network with three supplying firms producing with either coal, which is relatively cheap and polluting, combined cycle gas turbine (CCGT), which is the cleanest available conventional technology we consider, and open cycle gas turbine (OCGT), which has the lowest investment cost. A key result is that under demand uncertainty we see broader technology mixtures than in the deterministic case. OCGT is in the mixture as the peak load technology, whereas coal and CCGT are used as base load technologies. When a cap-and-trade system is implemented, we see that the tighter the allowance level, the more coal is replaced by the cleaner CCGT. In case of a fixed tax we observe that for low levels mainly coal is used, up to a certain threshold tax for which we see a shift to CCGT. The implications of limited transmission capacity in combination with government regulation are the following. In case of a maximum emission allowance level, limited transmission capacity may induce cleaner mixtures in case emission allowances are scarce. However, it does not necessarily induce investments in cleaner technologies, since limited transmission capacity may block such investments. Hence, investments in network capacity may be necessary to achieve the goal of motivating investment in cleaner technologies. In case of a fixed tax per unit emission we find that for higher tax levels a dirty technology is replaced by a cleaner technology. The network capacity may put a limit on this replacement. Therefore, it may be necessary to invest in network capacity in order to curb CO_2 levels.

Finally, we establish a relation between the optimal outcome in case of cap-and-trade and the optimal outcome in case of taxation. We show that for a special case of taxation, by taking the taxation level equal to the optimal price of emission allowances derived in the cap-and-trade model, we either find the same solution, or we find multiple solutions of which the cap-and-trade solution is one. In case there are multiple solutions, we see a trade-off between minimizing pollution and maximizing the regulator's surplus. Some of the solutions violate the maximum emission allowance level, indicating that when the optimal emission allowance price is set as a fixed tax, in the absence of a cap-and-trade system there is no way to enforce firms to choose the solution with minimal pollution.

The rest of the chapter is organized as follows. We begin with the introduction of the two-stage investment model with deterministic exogenous demand in Section 2.2. We sequentially reduce it to a single optimization problem. Section 2.3 introduces a stylized version of the model. We then derive our main findings concerning the effects of government regulations in the deterministic setting. In addition, an algorithm that finds the optimal mixture of technologies in case of a maximum emission allowance level is developed. In Section 2.4 we introduce the model with stochastic demand. A numerical study is presented in Section 2.5. Section 2.6 concludes.

2.2 The Investment Model - Deterministic Exogenous Demand

We deal with a perfectly competitive electricity market with deterministic exogenous demand; under perfect competition firms cannot exert market power and act as price takers. We discuss the basics of the electricity market and give an overview of the electricity market investment model as presented in Gürkan et al. (2013). Then, as an extension, we include either an emission constraint or a fixed carbon tax imposed by the environmental regulator.

The electricity market consists of a grid of supply and demand nodes connected by transmission lines. Typical to such an electricity network, compared to other networks treated in the literature, is first of all the non-storability of electricity. Secondly, power transmission between two nodes affects the capacities on all transmission lines in the network in accordance with Kirchhoff's voltage law; see for example Chao et al. (2000). At supply nodes, electricity producing firms are located, whereas consumers with a fixed exogenous demand are located at demand nodes. Decisions in the market are taken in two stages. At the first stage firms maximize their profits while choosing for each of their supply nodes the production capacities in the available technologies. All firms are assumed to take these investment decisions simultaneously without knowing the decisions of other firms and their effect on the electricity price. First stage profits depend on the equilibrium outcome of the second stage, where prices and production quantities are determined. At the second stage, firms determine their optimal production quantity given their investment capacities from the first stage as to maximize their second stage profits. In addition, a transmission system operator (TSO) owning the electricity grid is taking care of the transmission of power, while maximizing its own profits.

Finally, the market is cleared by means of market clearing conditions imposing that in each node supply of electricity should cover demand, and imposing a price cap in case of demand curtailment.

In addition, there is an environmental regulator that tries to curb and prevent high levels of CO_2 emissions in the electricity market. We focus on two main financial instruments available to such a regulator. One option is to impose a maximum allowance level for the total emissions. For each unit of CO_2 firms emit they should posses an allowance. Allowances can be traded on a secondary market. The "correct" price of an allowance depends on the number of allowances available and the demand for allowances. As soon as the total emissions hit the maximum allowed level, the price of the allowance becomes positive; otherwise it is zero. An alternative way for reducing the total emissions is to impose a fixed carbon tax per unit emission. With both instruments, the environmental regulator aims to reduce the production by the more polluting technologies and motivate firms to produce more with cleaner technologies, eventually inducing higher investment levels in cleaner technologies.

We call the resulting two-stage game containing all firms, the TSO, the market clearing conditions, and the environmental regulator, a perfect competition equilibrium problem. “Equilibrium” emphasizes that the firms optimize their own objective functions, and that their individual optimization problems are tied together by the market clearing conditions. When we are at a so-called perfectly competitive equilibrium, for none of the firms it is profitable to deviate.

A suitable mathematical framework for modeling the perfectly competitive electricity market is presented in Gürkan et al. (2013). However, that model does not include an environmental regulator or CO_2 regulation. In the next section, we introduce the two-stage game as presented in Gürkan et al. (2013) and incorporate the environmental regulator’s problem. As shown in Gürkan et al. (2013), the original two-stage game can be reduced to a single optimization problem. The same reduction can be done when adding a cap-and-trade system or imposing taxation, as we will show in Section 2.2.2.

2.2.1 Introducing the Two-Stage Game

The sets, parameters, and variables to be used in the perfect competition equilibrium problem are given below.

Sets:

N	:	the set of demand nodes
G	:	the set of firms
I_g	:	the set of supply nodes of firm $g \in G$
I	:	the set of all supply nodes $(I := \cup_g I_g)$
K_g	:	the set of technologies of firm $g \in G$
K	:	the set of all technologies $(K := \cup_g K_g)$
L	:	the set of electricity transmission lines connecting nodes in the network.

Parameters:

c_{ik}^g	:	unit production cost of firm g at supply node $i \in I_g$ for technology $k \in K_g$
κ_{ik}^g	:	unit investment cost of firm $g \in G$ at supply node $i \in I_g$ for technology $k \in K_g$
d_n	:	demand at demand node $n \in N$
$PTDF_{l,j}$	:	power transmitted through line $l \in L$ due to one unit of power injection into node $j \in N \cup I$
h_l	:	capacity limit of line $l \in L$
$VOLL$	:	value of unserved energy or lost load
E	:	total CO_2 emission allowed by the environmental regulator
e_k	:	units of CO_2 emitted per unit production with technology $k \in K_g$.

Variables:

x_{ik}^g	:	generation capacity investment of firm $g \in G$ for technology $k \in K_g$ at supply node $i \in I_g$
y_{ik}^g	:	quantity of power generated by firm $g \in G$ at supply node $i \in I_g$ by using technology $k \in K_g$
f_j	:	net power flow dispatched by the TSO to node $j \in N \cup I$
δ_j	:	unserved demand at node $j \in N \cup I$
p_j	:	electricity price at node $j \in N \cup I$
μ	:	unit allowance price.

For $g \in G$ we write $x^g = (x_{ik}^g)_{i \in I_g, k \in K_g}$ and $y^g = (y_{ik}^g)_{i \in I_g, k \in K_g}$, the vectors containing investment and production quantities, respectively, of firm g in all its technologies in all its supply nodes.

We next introduce the first and second stage problems. Each firm owns a set of technologies in some of its supply nodes. At the first stage, each firm $g \in G$ simultaneously decides on its optimal investment quantities x^g in all its available technologies in all its supply nodes. Per unit investment in technology $k \in K_g$ in supply node $i \in I_g$, firm $g \in G$ pays investment cost κ_{ik}^g. Firms determine their optimal investment quantities by maximizing their optimal second stage profits that are dependent on their investment quantities, minus their first stage investment costs. The second stage profit per unit production of firm $g \in G$ with technology $k \in K_g$ in supply node $i \in I_g$ consists of the market price of electricity, p_i, minus the unit production cost c_{ik}^g, and minus the price paid for emission allowances, $e_k\mu$. The optimization problem for firm $g \in G$ at stage one is

$$\max_{x^g \geq 0} \sum_{i \in I_g} \sum_{k \in K_g} (p_i - c_{ik}^g - e_k\mu) y_{ik}^g(x^g) - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_{ik}^g x_{ik}^g. \tag{2.1}$$

Here $y_{ik}^g(x^g)$ is the optimal production quantity of firm g at supply node $i \in I_g$ with technology $k \in K_g$ at the second stage if x^g is the investment quantity. p_i, $i \in I$, and μ are taken as parameters, since firms are behaving as price takers and are not aware that by changing their investments they may influence the price of electricity and the allowance price. Both will be determined at the second stage as a result of market clearing conditions and the emission allowance constraint, as we explain below.

At the second stage, firms treat the investment quantities as parameters. Each firm $g \in G$ determines its production quantities y^g by optimizing

$$\begin{aligned} \Pi_g(x^g) := \max_{y^g \geq 0} \quad & \sum_{i \in I_g} \sum_{k \in K_g} (p_i - c_{ik}^g - e_k\mu) y_{ik}^g \\ \text{s.t.} \quad & y_{ik}^g \leq x_{ik}^g \qquad\qquad (\beta_{ik}^g) \quad \forall i \in I_g, k \in K_g, \end{aligned} \tag{2.2}$$

where β_{ik}^g is the shadow price associated with the capacity constraint, representing the scarcity rent of technology $k \in K_g$ at supply node $i \in I_g$. The produced power is then dispatched by the transmission system operator (TSO) from the supply nodes to demand nodes. The TSO maximizes its profit from transmitting power while taking into account the network capacity. The electricity network consists of a set of transmission lines L in which each line $l \in L$ runs from one node to another node. The amounts transmitted, that is, the net flows into or out of each node $j \in N \cup I$, are denoted by f_j. $f_j > 0$ represents a flow into node j, whereas $f_j < 0$ represents a flow out of node j. For each unit of power flow into node j, some amount of power, given by the coefficient $PTDF_{l,j}$, is transmitted along transmission line $l \in L$. A power injection in one node typically affects the flows on all transmission lines (either positively or negatively). h_l is the capacity on transmission line $l \in L$, and total net power flow (which can be either negative or positive) on each line $l \in L$ must be between $-h_l$ and h_l. If there is limited capacity on some lines (h_l is finite for some $l \in L$), we call the network a capacitated network. The TSO's problem is formulated as follows:

$$\begin{aligned} \max_{f} \quad & \sum_{j \in N \cup I} p_j f_j \\ \text{s.t.} \quad & \sum_{j \in N \cup I} f_j = 0 && (\rho) \\ & h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 && (\lambda_l^+) \quad \forall l \in L \\ & h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 && (\lambda_l^-) \quad \forall l \in L. \end{aligned} \tag{2.3}$$

The first constraint, with corresponding dual variable ρ, is the flow balance constraint; that is, the total amount the TSO buys from firms should equal the total amount the TSO dispatches to demand nodes. The second and third constraints take into account the limited positive and negative transmission capacity in the network and have dual variables λ_l^+, $l \in L$, and λ_l^-, $l \in L$, respectively.

The environmental regulator determines the level of maximum emission allowance, E, which should be satisfied by the entire market and which is announced to the firms in advance. The price of emission allowances, μ, is then determined by the market. As long as the maximum allowance level is not reached, the price of an emission allowance will be zero; as soon as emissions hit the ceiling, μ will become positive as to create an incentive for firms to switch to cleaner technologies. This leads to the following complementarity condition:

$$0 \leq E - \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} e_k y_{ik}^g \quad \perp \quad \mu \geq 0. \tag{2.4}$$

Finally, there are two types of market clearing conditions. The market price of electricity is determined by the first type, which balances supply and demand in each node. Since not all firms are necessarily producing in all nodes, we define $G_j = \{g \in G | j \in I_g\}$, $j \in N \cup I$, as the set of firms that are active in node j. In each demand node $n \in N$, where production is zero, the net flow f_n into the node should be at least equal to the demand d_n, unless there is unsatisfied demand. In each supply node $i \in I$, where demand is defined as

$d_i = 0$, the net flow f_i will in general be negative, meaning it is a flow out of node i. This flow can be at most equal to the total production in node i. Hence, for each node $j \in N \cup I$ we have a constraint on the flows, balancing supply and demand. Nodal prices are determined perpendicular to each of these constraints. A second type of condition puts a cap on the price in each node and is known as VOLL pricing in the literature; see, for example, Stoft (2002), or alternatively Ehrenmann and Smeers (2008). Whenever demand cannot be satisfied at node $j \in N \cup I$, unsatisfied demand δ_j will be positive. Then, the price of electricity at node j is set at $VOLL$, the value of lost load. $VOLL$ is a relatively large number; that is, in numerical experiments it is typical to assume $VOLL = 10.000$, whereas the regular nodal prices usually lie between 30 and 80. The market clearing conditions are:

$$\begin{aligned} 0 \leq \sum_{g \in G_j} \sum_{k \in K_g} y^g_{jk} + \delta_j + f_j - d_j \quad &\perp \quad p_j \geq 0 \quad \forall j \in N \cup I \\ 0 \leq VOLL - p_j \quad &\perp \quad \delta_j \geq 0 \quad \forall j \in N \cup I. \end{aligned} \tag{2.5}$$

An alternative way of reducing the amount of CO_2 emitted by energy companies, is to tax firms per unit emission without imposing a bound on the total amount of CO_2 emitted. In such a system the environmental regulator fixes a level of taxation, say $\bar{\mu}$. In the model the (optimal) price of emission allowances, μ, should be replaced by the fixed parameter $\bar{\mu}$ and the emission constraint (2.4) should be omitted. In the next section we analyze the two models in further detail.

2.2.2 Reduction to a Single Optimization Problem

In Gürkan et al. (2013), it is shown that the problem of finding a perfect competition equilibrium, excluding the emission constraint (2.4), between the firms and the TSO can be written as a single optimization problem. This result can be extended to the models which include either an emission constraint or taxation imposed by the environmental regulator. We do not discuss the derivations in detail here (see Gürkan et al. (2013)); however, we briefly elaborate on the results that are relevant to us.

First we show that there exists a single optimization problem that simultaneously solves the firms' second stage problems (2.2) and the TSO's second stage problem (2.3), under the emission constraint (2.4) and the market clearing conditions (2.5). As a result, we are left with a single optimization problem at the second stage. Then we show that the first and the second stage problems together can be written as a single optimization problem. Having been reduced to a single optimization problem, a perfect competition equilibrium can be easily found by using standard optimization techniques.

In order to show that there exists a single optimization problem that solves all second stage problems simultaneously, we first write the KKT optimality conditions for all the second stage problems introduced in Section 2.2.1 for given $x = (x^g)_{g \in G}$:

$$
\begin{array}{rcll}
0 \leq \beta_{ik}^{*g} - p_i^* + c_{ik}^g + e_k \mu^* & \perp & y_{ik}^{*g} \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
0 \leq x_{ik}^g - y_{ik}^{*g} & \perp & \beta_{ik}^{*g} \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
0 \leq VOLL - p_j^* & \perp & \delta_j^* \geq 0 & \forall j \in N \cup I \\
0 \leq \sum_{g \in G_j} \sum_{k \in K_g} y_{jk}^{*g} + \delta_j^* + f_j^* - d_j & \perp & p_j^* \geq 0 & \forall j \in N \cup I \\
0 \leq h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j^* & \perp & \lambda_l^{*+} \geq 0 & \forall l \in L \\
0 \leq h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j^* & \perp & \lambda_l^{*-} \geq 0 & \forall l \in L \\
0 \leq E - \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} e_k y_{ik}^{*g} & \perp & \mu^* \geq 0 & \\
\sum_{l \in L} PTDF_{l,j}(\lambda_l^{*+} - \lambda_l^{*-}) + & & & \\
p_j^* - \rho^* = 0 & & & \forall j \in N \cup I \\
\sum_{j \in N \cup I} f_j^* = 0. & & &
\end{array}
\tag{2.6}
$$

A point which satisfies these KKT conditions (2.6) also solves (2.2) and (2.3) under the constraints (2.4) and (2.5).

The KKT conditions (2.6) correspond to a single optimization problem, referred to as the Optimal Power Flow Problem (OPF):

$$
\begin{aligned}
Z(x) :=\\
\min_{y,f,\delta} \quad & \sum_{g\in G}\sum_{i\in I_g}\sum_{k\in K_g} c^g_{ik} y^g_{ik} + VOLL \sum_{j\in N\cup I} \delta_j \\
\text{s.t.} \quad & \sum_{g\in G_j}\sum_{k\in K_g} y^g_{jk} + \delta_j + f_j \ge d_j && (p_j) \quad \forall j \in N\cup I \\
& \sum_{j\in N\cup I} f_j = 0 && (\rho) \\
& h_l - \sum_{j\in N\cup I} PTDF_{l,j} f_j \ge 0 && (\lambda^+_l) \quad \forall l \in L \\
& h_l + \sum_{j\in N\cup I} PTDF_{l,j} f_j \ge 0 && (\lambda^-_l) \quad \forall l \in L \\
& x^g_{ik} - y^g_{ik} \ge 0 && (\beta^g_{ik}) \quad \forall g\in G, i\in I_g, k\in K_g \\
& E - \sum_{g\in G}\sum_{i\in I_g}\sum_{k\in K_g} e_k y^g_{ik} \ge 0 && (\mu) \\
& y^g_{ik} \ge 0 && \forall g\in G, i\in I_g, k\in K_g \\
& \delta_j \ge 0 && \forall j\in N\cup I.
\end{aligned}
\tag{2.7}
$$

The basic idea of this derivation was originally introduced in Boucher and Smeers (2001) for a game between firms, consumers, and TSO, and can be used in more general settings such as the one we consider here. Solving the OPF problem clearly results in the optimal solution for the firms' problems at the second stage, the TSO's problem, and the environmental regulator's problem. The resulting optimal solution is a perfectly competitive equilibrium of the market at the second stage, since none of the players will have an incentive to deviate.

Next, we briefly outline how the problem in stage one, consisting of optimization problem (2.1) for each firm, together with the OPF problem (2.7) at stage two, can be written as a single optimization problem. When demand is assumed to be deterministic, Lemma 1.1 of Gürkan et al. (2013) states that the optimal investment amount x^{*g}_{ik} is equal to the optimal production amount y^{*g}_{ik} for all $g \in G$, $i \in I_g$, $k \in K_g$, and Lemma 1.2 of Gürkan et al. (2013) states that for x^{*g}_{ik} to be positive at the equilibrium, β^{*g}_{ik} should be equal to κ^g_{ik}. These results are employed in Lemma 1.3 of Gürkan et al. (2013) to show that a point satisfying a particular set of KKT conditions is an equilibrium solution of the two-stage game consisting of (2.1) and (2.7). When we use the corresponding result in our setting, the resulting set of KKT conditions is the following:

$$
\begin{aligned}
0 \leq \kappa_{ik}^{g} - p_i^* + c_{ik}^{g} + e_k \mu^* \quad &\perp \quad x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g \\
0 \leq VOLL - p_j^* \quad &\perp \quad \delta_j^* \geq 0 \quad \forall j \in N \cup I \\
0 \leq \sum_{g \in G_j} \sum_{k \in K_g} x_{jk}^{*g} + \delta_j^* + f_j^* - d_j \quad &\perp \quad p_j^* \geq 0 \quad \forall j \in N \cup I \\
0 \leq h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j^* \quad &\perp \quad \lambda_l^{*+} \geq 0 \quad \forall l \in L \\
0 \leq h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j^* \quad &\perp \quad \lambda_l^{*-} \geq 0 \quad \forall l \in L \\
0 \leq E - \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} e_k x_{ik}^{*g} \quad &\perp \quad \mu^* \geq 0 \\
\sum_{l \in L} PTDF_{l,j}(\lambda_l^{*+} - \lambda_l^{*-}) + p_j^* - \rho^* = 0 \quad & \quad \forall j \in N \cup I \\
\sum_{j \in N \cup I} f_j^* = 0. &
\end{aligned}
$$

This set of KKT conditions corresponds to the following single optimization problem:

$$
\begin{aligned}
\min_{x,f,\delta} \quad & \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} (c_{ik}^{g} + \kappa_{ik}^{g}) x_{ik}^{g} + VOLL \sum_{j \in N \cup I} \delta_j \\
\text{s.t.} \quad & \sum_{g \in G_j} \sum_{k \in K_g} x_{jk}^{g} + \delta_j + f_j \geq d_j \quad (p_j) \quad \forall j \in N \cup I \\
& \sum_{j \in N \cup I} f_j = 0 \quad (\rho) \\
& h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 \quad (\lambda_l^{+}) \quad \forall l \in L \\
& h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 \quad (\lambda_l^{-}) \quad \forall l \in L \\
& E - \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} e_k x_{ik}^{g} \geq 0 \quad (\mu) \\
& x_{ik}^{g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g \\
& \delta_j \geq 0 \quad \forall j \in N \cup I.
\end{aligned}
\tag{2.8}
$$

Note that (2.8) simultaneously solves the optimization problems (2.1), (2.2), and (2.3), while taking the emission constraint (2.4) and the market clearing conditions (2.5) into account. As a consequence, the optimal solution (x^*, δ^*, f^*) of (2.8) results in optimality for all firms at both stages.

We now turn our attention to the fixed tax model. As mentioned earlier, one needs to replace the (optimal) price of emission allowances μ by the fixed parameter $\bar{\mu}$, and omit the emission constraint. It then turns out that the two-stage game can again be written as a single optimization problem for this model. Without going into details, the resulting optimization problem is:

$$\begin{aligned}
\min_{x,f,\delta} \quad & \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} (c_{ik}^g + \kappa_{ik}^g + e_k \bar{\mu}) x_{ik}^g + VOLL \sum_{j \in N \cup I} \delta_j \\
\text{s.t.} \quad & \sum_{g \in G_j} \sum_{k \in K_g} x_{jk}^g + \delta_j + f_j \geq d_j \quad (p_j) \quad \forall j \in N \cup I \\
& \sum_{j \in N \cup I} f_j = 0 \quad (\rho) \\
& h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 \quad (\lambda_l^+) \quad \forall l \in L \\
& h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 \quad (\lambda_l^-) \quad \forall l \in L \\
& x_{ik}^g \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g \\
& \delta_j \geq 0 \quad \forall j \in N \cup I.
\end{aligned} \tag{2.9}$$

2.3 Equilibrium Analysis

The qualitative equilibrium analysis of the underlying two-stage game is obscured by the intractability of the problem in its current form because of the network effects due to the underlying network topology, the transmission line capacities, and the associated PTDFs. In order to understand the direct effects of the emission constraint or taxation on the decisions of the firms, we work with the following simplifying assumptions. There is a single node with a given demand d and n firms. Each firm possesses a single and unique technology in K. Hence, we compromise indices g, i, and k by simply k and denote by $K = \{1, ..., n\}$ the set of all firms or technologies. In effect, having a single node means that we are focusing on a network with unlimited transmission capacities and that we can omit the flows and the PTDF-constraints. In addition, we write $a_k := c_k + \kappa_k$, the total cost per unit production in technology $k \in K$ and make the following assumption on the parameters:

$$a_1 < a_2 < \cdots < a_n \qquad \text{and} \qquad e_1 > e_2 > \cdots > e_n.$$

That is, the technologies are ordered such that technology 1 is the cheapest and the dirtiest, and technology n is the most expensive and the cleanest technology. This assumption is reasonable, since dirty technologies typically have relatively low long-run marginal cost, but have a relatively high level of CO_2

emission per unit production; later we will argue that this assumption can in fact be made without loss of generality, see Remark 1.

Let s be the slack variable in the emission constraint. The stylized version of model (2.8), with only one node and hence no network effects, is summarized as:

$$\begin{array}{llllll}
\min\limits_{x,\delta,s} & \sum\limits_{k=1}^{n} a_k x_k & +VOLL\delta & & & \\
\text{s.t.} & \sum\limits_{k=1}^{n} x_k & +\delta & \geq & d & (p) \\
 & -\sum\limits_{k=1}^{n} e_k x_k & -s & = & -E & (\mu) \\
 & x_k & & \geq & 0 & \forall k \in K \\
 & & \delta, \quad s & \geq & 0. &
\end{array} \tag{2.10}$$

Note that minus signs appear on both sides of the emission constraint. Obviously, it is possible to multiply this constraint by -1, but this results in a non-positive dual variable. Since that will be more tricky to interpret, we will work with the current formulation (2.10). This way, the dual variable actually gives the "correct" price of the unit emission allowance.

The stylized version of the fixed tax model (2.9) is obtained in a similar way:

$$\begin{array}{llllll}
\min\limits_{x,\delta} & \sum\limits_{k=1}^{n} (a_k + e_k\bar{\mu}) x_k & +VOLL\delta & & & \\
\text{s.t.} & \sum\limits_{k=1}^{n} x_k & +\delta & \geq & d & (p) \\
 & x_k & & \geq & 0 & \forall k \in K \\
 & & \delta & \geq & 0. &
\end{array} \tag{2.11}$$

Next, we elaborate on the perfect competition equilibrium in both models (2.10) and (2.11). In Section 2.3.1, we derive that in case of a maximum allowance level one or two firms will be producing at the equilibrium. This will have an important consequence. When there is enough capacity in the system, different demand levels will impose different prices for a unit of emission allowance and there will not be a fixed merit order of the technologies. An algorithm for finding the optimal mixture in the equilibrium in a systematic way is proposed in Section 2.3.2. A proof showing that the algorithm finds the optimal solution, is included in the Appendix (Section 2.7). In case of a fixed tax, there will be a fixed merit order and only one firm will be producing at the equilibrium; this and its implications are dealt with in Section 2.3.3. Section

2.3.4 provides a characterization of the technologies that can never be the first in the merit order; hence those technologies will permanently be dominated by other technologies. Finally, Section 2.3.5 analyzes the effects of cap-and-trade and taxation on consumer prices via the concept of consumers' surplus.

2.3.1 Characterizing the Equilibrium with a Cap on Total Emissions

As there are only two constraints in the stylized model (2.10), there will be only two basic variables. Therefore, at most two technologies will be producing at the equilibrium. Obviously, without an emission constraint (that is, when $E = \infty$) only the cheapest technology will be used. On the other hand, when the emission constraint is very tight (that is, when E is extremely low), only the cleanest technology can be used. In all other cases, two technologies, a relatively cheap one and a relatively clean one, will be contained in the optimal mixture.

In order to find which technologies will be part of the optimal mix, we divide the set of firms into two sets; the first set, J, contains the relatively cheap and dirty firms, whereas the second set, H, contains the remaining, relatively expensive and clean, firms. To be more precise, a firm is in J when it cannot satisfy the demand on its own without violating the emission constraint; a firm is in H otherwise. We must then find the cheapest combination of a firm in J and a firm in H, which can together satisfy the emission allowance constraint. Note that, when $J = N$, even the cleanest firm cannot satisfy demand without violating the emission constraint. On the other hand, when $H = N$, all demand can be produced by the cheapest firm without violating the emission constraint. The following proposition summarizes these results formally and lays down a property that the basic variables of the linear program (2.10) should satisfy at optimality when J and H are both nonempty.

Proposition 2.1. *Suppose that there are n firms for which it holds that*

$$a_1 < a_2 < \cdots < a_n \qquad \text{and} \qquad e_1 > e_2 > \cdots > e_n.$$

Given E and d, define

$$J = \{1, ..., i\} \qquad \text{and} \qquad H = \{i+1, ..., n\},$$

where $i \in K \cup \{0\}$ is such that $e_{i+1}d \leq E < e_i d$ (with $e_0 = \infty$ and $e_{n+1} = 0$). Then, at the perfect competition equilibrium, that is, at the optimal solution to (2.10), *exactly one of the following cases holds:*

(i) All firms are able to satisfy the demand without violating the emission constraint. This is the case when $J = \emptyset$. *At the equilibrium,* $x_1^* = d$, $x_k^* = 0$ *for* $k = 2, ..., n$, $\delta^* = 0$, *and* $s^* = E - e_1 d$.

(ii) No firm is able to satisfy the demand without violating the emission constraint. This is the case when $H = \emptyset$. *At the equilibrium,* $x_n^* = E/e_n$, $x_k^* = 0$ *for* $k = 1, ..., n-1$, $\delta^* = d - \frac{E}{e_n}$, *and* $s^* = 0$.

(iii) A combination of a relatively clean and a relatively dirty firm will satisfy demand without violating the emission constraint. This is the case when both J *and* H *are non-empty. Firms* $j \in J$ *and* $h \in H$ *produce at the equilibrium if* j *and* h *satisfy*

$$a_k(e_j - e_h) + a_j(e_h - e_k) + a_h(e_k - e_j) \geq 0 \quad \forall k \in \{1, ..., n\}. \tag{2.12}$$

Moreover, it holds that at the equilibrium $x_j^* = \frac{E - e_h d}{e_j - e_h}$, $x_h^* = \frac{e_j d - E}{e_j - e_h}$, $x_k^* = 0$ *for* $k \neq j, h$, $\delta^* = 0$, $s^* = 0$.

Proof: Let p be the price of electricity and μ the price of unit emission. That is, p and μ are the dual variables associated with the first and second constraint of (2.10). Then we can write the KKT optimality conditions of the linear program (2.10) as

$$\begin{aligned} 0 \leq a_k - p^* + e_k\mu^* \quad &\perp \quad x_k^* \geq 0 \quad \forall k \in \{1, ..., n\} \\ 0 \leq VOLL - p^* \quad &\perp \quad \delta^* \geq 0 \\ 0 \leq \sum_{k=1}^{n} x_k^* + \delta^* - d \quad &\perp \quad p^* \geq 0 \\ 0 \leq E - \sum_{k=1}^{n} e_k x_k^* \quad &\perp \quad \mu^* \geq 0. \end{aligned} \tag{2.13}$$

If $J = \emptyset$, then $e_1 d \leq E$ and therefore x_1 and s are the basic variables in (2.10) with basis

$$c_B = \begin{bmatrix} a_1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ -e_1 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} d \\ -E \end{bmatrix}.$$

Hence,

$$[p^* \quad \mu^*] = c_B B^{-1} = [a_1 \quad 0].$$

The optimal solution is $x_1^* = d$, $x_k^* = 0$ for $k = 2, ..., n$, $\delta^* = 0$, and $s^* = E - e_1 d$.

This is case (i).

If $H = \emptyset$, then $e_n d > E$ and therefore x_n and δ are the basic variables in (2.10) with basis

$$c_B = \begin{bmatrix} a_n & VOLL \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ -e_n & 0 \end{bmatrix}, \quad b = \begin{bmatrix} d \\ -E \end{bmatrix}.$$

Hence,

$$[p^* \quad \mu^*] = c_B B^{-1} = [VOLL \quad \frac{VOLL - a_n}{e_n}].$$

The optimal solution is $x_n^* = E/e_n$, $x_k^* = 0$ for $k = 1, ..., n-1$, $\delta^* = d - \frac{E}{e_n} > 0$, and $s^* = 0$. This is case (ii).

If $J \neq \emptyset$ and $H \neq \emptyset$, then for some $j \in J$, $h \in H$, x_j and x_h are the basic variables in (2.10) with basis

$$c_B = \begin{bmatrix} a_j & a_h \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ -e_j & -e_h \end{bmatrix}, \quad b = \begin{bmatrix} d \\ -E \end{bmatrix}.$$

Hence,

$$[p^* \quad \mu^*] = c_B B^{-1} = \begin{bmatrix} \frac{-a_j e_h + a_h e_j}{e_j - e_h} & \frac{a_h - a_j}{e_j - e_h} \end{bmatrix}.$$

The optimal solution is $x_j^* = \frac{E - e_h d}{e_j - e_h}$, $x_h^* = \frac{e_j d - E}{e_j - e_h}$, $x_k^* = 0$ for $k \neq j, h$, $\delta^* = 0$, $s^* = 0$. This is case (iii). The solution found is a perfect competition equilibrium when (2.12) holds, since this implies that the first condition of (2.13) is satisfied, that is,

$$0 \leq a_k - \frac{-a_j e_h + a_h e_j}{e_j - e_h} + \frac{a_h - a_j}{e_j - e_h} e_k = a_k - p^* + e_k \mu^* \quad \forall k \in \{1, ..., n\}.$$

One can easily check that the other conditions in (2.13) are also satisfied, by noting that $j \in J$, $h \in H$, and $e_h d \leq E < e_j d$ holds. $\square$

Since determining the sets J and H is straightforward, it is easy to see which case in Proposition 2.1 holds. However, finding a $j \in J$ and $h \in H$ in case (iii) that satisfy (2.12) is not straightforward. In Section 2.3.2 we give an algorithm for finding the optimal $j \in J$ and $h \in H$. First, we provide an interpretation of the optimal price of emission allowances and the optimal electricity price.

The proof of Proposition 2.1 shows that it is possible to derive both μ^* and p^* explicitly at optimality. One can interpret μ^* as follows. In case (i), μ^* is zero, since one should not pay for an extra unit of CO_2 emission when the emission constraint is not active. In case (ii), since demand is not satisfied, $VOLL$ pricing is in effect, and consequently μ^* is relatively high. When E decreases, one produces less with technology n and the unsatisfied demand δ^* increases. Let Δy denote the change in unsatisfied demand and ΔE denote the change in E. Since the new emission constraint should be satisfied, we need to have

$$-e_n \Delta y = \Delta E.$$

Therefore,

$$\Delta y = -\frac{\Delta E}{e_n}.$$

Then, one would pay $\Delta y \cdot VOLL$ more and save $\Delta y \cdot a_n$. If we take $\Delta E = -1$, then μ^* gives the marginal cost for satisfying a lower emission allowance level, as long as the optimal basis does not change. In other words, μ^* is equal to the price one pays to substitute the production of firm n by unsatisfied demand for a unit decrease in E:

$$\mu^* = (VOLL - a_n)\Delta y = \frac{VOLL - a_n}{e_n}.$$

In case (iii), μ^* represents the price to pay to substitute the dirty technology by the clean technology when E decreases. Let Δy denote the extra amount of the clean technology that is produced. This volume Δy of the substitution of the dirty technology by the clean technology should satisfy

$$-e_j \Delta y + e_h \Delta y = \Delta E,$$

in order to satisfy the emission constraint. $e_j \Delta y$ is the amount of CO_2 emission saved and $e_h \Delta y$ is the amount of CO_2 emitted instead. Hence, we get

$$\Delta y = \frac{\Delta E}{e_h - e_j}.$$

Due to the lower allowance level, one saves $a_j \Delta y$ and additionally pays $a_h \Delta y$. Again, if we take $\Delta E = -1$, then μ^* represents the price to pay for substituting dirty technology by clean technology as a result of a unit decrease in E:

$$\mu^* = (a_h - a_j)\Delta y = \frac{a_h - a_j}{e_j - e_h}.$$

A similar argument holds for p^* when we perturb demand d. In case (i), firm 1 can satisfy demand without violating the emission constraint. As long as the optimal basis does not change, a unit increase of demand costs $p^* = a_1$. In case (ii), demand is not satisfied; hence, $VOLL$ pricing is in effect and an extra unit of demand causes the unsatisfied demand to increase by one unit. The extra cost involved equals $p^* = VOLL$. In case (iii), p^* represents the extra cost incurred (shared by firms j and h) in order to satisfy an extra unit of demand. In order to do so, they should take care that, with the new production amounts, the total emission does not increase; that is,

$$e_j(\Delta d - \Delta y) + e_h \Delta y = 0,$$

where Δy denotes the extra production by firm h, whereas Δd denotes the extra demand. Hence $\Delta d - \Delta y$ represents the change in firm j's production. Rewriting gives

$$\Delta y = \frac{\Delta d e_j}{e_j - e_h}.$$

Furthermore, when demand increases with Δd, and given that the basis does not change, the total production cost changes by $a_h \Delta y$ and $a_j(\Delta d - \Delta y)$. If we take $\Delta d = 1$ then

$$p^* = a_j(1 - \Delta y) + a_h \Delta y = \frac{a_h e_j - a_j e_h}{e_j - e_h}.$$

2.3.2 Algorithms for Finding the Equilibrium

Given a maximum emission allowance level E and demand d, we determined sets of technologies $J = \{1, ..., i\}$ and $H = \{i+1, ..., n\}$ where $i \in K \cup \{0\}$ is such that $e_{i+1}d \leq E < e_i d$. We next discuss two algorithms for finding $j \in J$ and $h \in H$ such that (2.12) is satisfied. Initially we generate candidate sets $\bar{J}$ and $\bar{H}$ for J and H, respectively. By starting with the smallest non-empty $\bar{J}$ or $\bar{H}$ and expanding or reducing the sets systematically we will reach to the optimal j and h in a finite number of iterations. In each iteration, we find a $j^* \in \bar{J}$ and an $h^* \in \bar{H}$ such that (2.12) is satisfied. Afterwards, we evaluate whether j^* and h^* belong to the sets J and H, respectively. If yes, the optimal solution is found. If not, the algorithm systematically expands one candidate set and reduces the other.

The forward version of the algorithm first checks whether J or H is empty. If so, we are in case *(i)* or *(ii)* of Proposition 2.1, respectively. If not, we let $j^* = 1$ and start with the smallest non-empty candidate set for J, $\bar{J} = \{1\}$. Since J is nonempty, firm 1 cannot satisfy all demand without violating the emission constraint. A cleaner and more expensive firm has to contribute. As a result, the total production cost will increase. The algorithm will then find $h^* \in \bar{H} = \{2, ..., n\}$ such that this increase in production cost is minimized. By doing this, we guarantee that (2.12) is satisfied, as shown in the proof that can be found in the appendix. If $h^* \in H$, the optimal solution is found. If not, we have to do another iteration. We first expand $\bar{J}$ by adding $\{j^* + 1, ..., h^*\}$ and reduce $\bar{H}$ accordingly. Then, we set $j^* = h^*$ and find again the firm in the new candidate set $\bar{H}$ that minimizes the extra cost. The algorithm terminates when $h^* \in H$; that is, we found a $j^* \in J$ and $h^* \in H$ such that (2.12) is satisfied. The optimal solution to (2.10) then immediately follows from Proposition 2.1. One may wonder why we pick $j^* = h^*$ in each new iteration. In the proof of the algorithm we show that if the new j^* is not h^*, then for at least one firm k inequality (2.12) is violated.

The backward version of the algorithm has the same structure. It first checks if J or H is empty. If not, we let $h^* = n$ and start with the smallest non-empty candidate set for H, $\bar{H} = \{n\}$. Since H is nonempty, firm n is able to satisfy all demand without violating the emission constraint. However, we can reduce the objective function value by letting a more polluting and cheaper firm contribute. The algorithm will find the firm $j^* \in \bar{J}$ that maximizes the reduction of the total cost. Again by ensuring this, (2.12) is automatically satisfied for all k. If $j^* \in H$, we can gain even more by letting another firm in $\bar{J} \backslash \{j^*, ..., h^* - 1\}$ contribute instead of h^*. We hence do another iteration and update $\bar{J}$ by removing $\{j^*, ..., h^* - 1\}$ and update $\bar{H}$ accordingly. Then we set $h^* = j^*$ and find the firm that maximizes the reduction of the total cost. The algorithm terminates when $j^* \in J$.

Algorithm 2.1. *The Forward Version*

Step 0:

Given E and d, let $J = \{1, ..., i\}$ and $H = \{i + 1, ..., n\}$, where $i \in K \cup \{0\}$ is such that $e_{i+1}d \leq E < e_i d$.

Step 1:
If $J = \emptyset$, STOP$= 1$, Output$= (1, 1)$.
Else if $H = \emptyset$, STOP$= 1$, Output$= (n, n)$.
Else STOP$= 0$, $j^* = 1$, $h^* = 1$.

Step 2:
While STOP$= 0$ do
Let $\bar{J} = \{1, ..., h^*\}$, $\bar{H} = N \backslash \bar{J}$, $j^* = h^*$, $h^* = \arg\min_{h \in \bar{H}} \{\frac{a_h - a_{j^*}}{e_{j^*} - e_h}\}$;

If $h^* \in H$, $j = j^*$, $h = h^*$, STOP$= 1$, Output$= (j, h)$;
end.

Algorithm 2.2. *The Backward Version*
Step 0:
Given E and d, let $J = \{1, ..., i\}$ and $H = \{i + 1, ..., n\}$, where $i \in K \cup \{0\}$ is such that $e_{i+1}d \leq E < e_i d$.

Step 1:
If $J = \emptyset$, STOP$= 1$, Output$= (1, 1)$.
Else if $H = \emptyset$, STOP$= 1$, Output$= (n, n)$.
Else STOP$= 0$, $j^* = n$, $h^* = n$.

Step 2:
While STOP$= 0$ do
Let $\bar{H} = \{j^*, ..., n\}$, $\bar{J} = N \backslash \bar{H}$, $h^* = j^*$, $j^* = \arg\max_{j \in \bar{J}} \{\frac{a_{h^*} - a_j}{e_j - e_{h^*}}\}$;

If $j^* \in J$, $j = j^*$, $h = h^*$, STOP$= 1$, Output$= (j, h)$;
end.

Given E, d, and the characteristics of the firms, we can apply either one of the algorithms to find the optimal (j, h). A proof showing that the Forward algorithm finds the optimal solution is included in the Appendix (Section 2.7); clearly one can write a proof for the Backward algorithm in a similar way. Also note that when Output$= (1, 1)$, firm 1 is the only producer (case (i) of Proposition 2.1) and when Output$= (n, n)$, firm n is the only producer (case (ii) of Proposition 2.1).

Remark 1. From the arguments in the appendix, it becomes clear that the assumption that $a_1 < a_2 < \cdots < a_n$ and $e_1 > e_2 > \cdots > e_n$ can in fact be made without loss of generality. If a technology would not obey this assumption, it would either dominate another technology or it would be dominated by another technology itself. A technology that is dominated by another technology will never be chosen by the algorithm since it is both more polluting and more expensive than the other one.

The following proposition establishes the monotonicity relationship between the price of electricity, the price of emission allowances, and the maximum emission allowance level.

Proposition 2.2. *The price of electricity p^* and the price of emission allowances μ^* are weakly increasing as E decreases.*

Proof: Given E, let (j_E, h_E) be the producing firms at the perfect equilibrium. We let E decrease to $\tilde{E} < E$ and consider the effect. Define

$$\tilde{J} = \{1, ..., \tilde{i}\} \qquad \text{and} \qquad \tilde{H} = \{\tilde{i}+1, ..., n\},$$

where $\tilde{i} \in K \cup \{0\}$ is such that $e_{\tilde{i}+1}d \leq \tilde{E} < e_{\tilde{i}}d$. Note that $\tilde{H} \subseteq H$; that is, since the emission constraint became tighter, we may need to choose a cleaner firm. Two things may happen.

(1) $h_E \in \tilde{H}$: Then (j_E, h_E) is still the optimal mixture. p^* and μ^* remain the same.

(2) $h_E \notin \tilde{H}$: Apply the Forward Algorithm with $j^* = h_E$, $\bar{J} = \{1, ..., h_E\}$, and $\bar{H} = N \backslash \bar{J}$ and find

$$h^* = \arg\min_{h \in \bar{H}} \{\frac{a_h - a_{h_E}}{e_{h_E} - e_h}\}.$$

In case this $h^* \in \tilde{H}$, the new optimal solution is found and we can compare the prices of allowances. First notice that, since j_E and h_E were producing at the equilibrium for the allowance level E, (2.12) holds for all k. In particular, for $k = h^*$ we have

$$a_{j_E}(e_{h_E} - e_{h^*}) + a_{h_E}(e_{h^*} - e_{j_E}) + a_{h^*}(e_{j_E} - e_{h_E}) \geq 0. \tag{2.14}$$

Using this, we compare the prices of allowances and the prices of electricity:

$$\mu^*_{\tilde{E}} - \mu^*_E = \frac{a_{h^*} - a_{h_E}}{e_{h_E} - e_{h^*}} - \frac{a_{h_E} - a_{j_E}}{e_{j_E} - e_{h_E}} =$$

$$\frac{a_{j_E}(e_{h_E} - e_{h^*}) + a_{h_E}(e_{h^*} - e_{j_E}) + a_{h^*}(e_{j_E} - e_{h_E})}{(e_{h_E} - e_{h^*})(e_{j_E} - e_{h_E})} \geq 0.$$

Since the denominator and the numerator are positive by the fact that $e_{h^*} < e_{h_E} < e_{j_E}$ and inequality (2.14), respectively, the last inequality holds. Similarly,

$$p^*_{\tilde{E}} - p^*_E = \frac{e_{h_E} a_{h^*} - e_{h^*} a_{h_E}}{e_{h_E} - e_{h^*}} - \frac{e_{j_E} a_{h_E} - e_{h_E} a_{j_E}}{e_{j_E} - e_{h_E}} =$$

$$e_{h_E} \frac{a_{j_E}(e_{h_E} - e_{h^*}) + a_{h_E}(e_{h^*} - e_{j_E}) + a_{h^*}(e_{j_E} - e_{h_E})}{(e_{h_E} - e_{h^*})(e_{j_E} - e_{h_E})} \geq 0.$$

In case $h^* \notin \tilde{H}$, continue the algorithm. Then, μ^* and p^* even further increase. □

2.3.3 Characterizing the Equilibrium with a Fixed Tax per Unit Emission

We have seen that there is no fixed merit order of technologies in (2.10). In order to satisfy the emission allowance, different technologies may be chosen in the optimal mixture for different levels of demand. In particular, for each demand realization we get a combination of a cheap but dirty technology and an expensive but clean technology; this combination depends on the level of demand. As a consequence, the entire industry is motivated to have several technologies available.

We now turn our attention to the situation where the environmental regulator taxes firms per unit emission. In the fixed tax model (2.11), there is a fixed merit order on the firms. Only the firm(s) with the lowest total marginal cost, being production cost plus investment cost plus tax on CO_2 emission, will be producing at the equilibrium. Hence, this system singles out one or, in case of equal marginal cost, several technologies and is independent of demand levels as long as there is sufficient capacity. In addition, we show the implications of taking the optimal price of emission allowances from the cap-and-trade model as the fixed tax. In this special case, the fixed tax model cannot guarantee that emissions stay below the maximum allowance level and hence may lead to a more polluting technology mixture.

To illustrate this result, suppose that, for given $\bar{\mu}$, we may reorder the firms such that

$$a_1 + \bar{\mu} e_1 \leq a_2 + \bar{\mu} e_2 \leq \cdots \leq a_n + \bar{\mu} e_n. \tag{2.15}$$

This is, aside from possible equalities, the fixed merit order of the firms in the presence of taxation. In order to illustrate our results in a clear way, we additionally assume that one or both of the first two inequalities in (2.15) are strict. At the optimal solution to (2.11), either firm 1 or both firm 1 and firm 2 are producing. Firm 1 is the only firm producing at the equilibrium when $a_1 + \bar{\mu} e_1 < a_2 + \bar{\mu} e_2$; both firms might be producing when equality holds, since then both are equally cheap. Hence, independent of the level of demand, we know which firm(s) will be producing at the equilibrium.

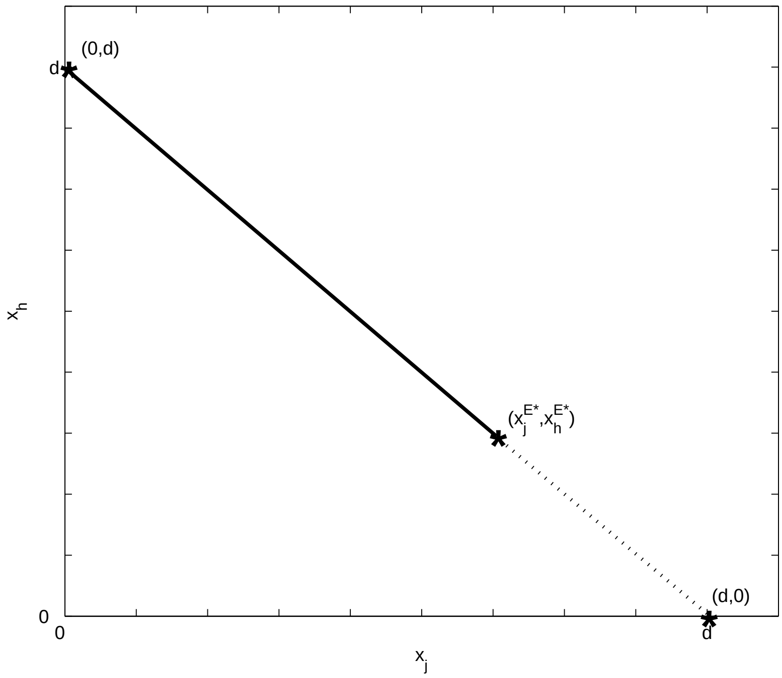
In order to see that we cannot necessarily guarantee that the total amount of CO_2 emitted will remain below some maximum allowance level, we will consider a special case of taxation, namely $\bar{\mu} = \mu^*$. In order to distinguish between the cap-and-trade and the taxation solution we will give the corresponding variables a superscript E and T, respectively. First, recall that in the cap-and-trade model, for given E, firms j and h, with j the relatively dirty but cheap firm and h the relatively clean but expensive firm, are found as producing firms at the equilibrium in (2.10). By Proposition 2.1, we have optimal production quantities $x_j^{E*} = \frac{E - e_h d}{e_j - e_h}$, $x_h^{E*} = \frac{e_j d - E}{e_j - e_h}$, $x_k^{E*} = 0$ for $k \neq j, h$, and

$$(p^{E*}, \mu^*) = \left(\frac{-a_j e_h + a_h e_j}{e_j - e_h}, \frac{a_h - a_j}{e_j - e_h} \right).$$

Now, we choose $\bar{\mu} = \mu^*$. Obviously, the optimal solution (x^{E*}, p^{E*}) is also an optimal solution to (2.11), since the set of KKT conditions to (2.11) is a subset of (2.13). Hence, firm j and firm h must be the first two firms in the merit order (2.15) and their effective marginal costs are equal, namely $a_j + \bar{\mu} e_j = a_h + \bar{\mu} e_h$. Therefore, any convex combination x^{T*} with (x_j^{T*}, x_h^{T*}) between $(d, 0)$ and $(0, d)$ is also a solution to the fixed tax problem (2.11); see Figure 2.1 where we draw the set of optimal solutions for firms j and h. In fact, there is a trade-off between producing with the cheaper technology and satisfying the emission constraint. The total emission will depend on this trade-off, and may exceed the maximum allowance level E. To see this, note that for the solution x^{T*} with $(x_j^{T*}, x_h^{T*}) = (d, 0)$, $e_j d$ is the total amount of CO_2 emitted; however, by the choice of j and h, we have $e_j d > E$. Hence, this solution would not have been allowed in (2.10). In particular, all points on the dotted line in Figure 2.1 would exceed the maximum allowance level, although they give optimal solutions of (2.11).

On the other hand, all solutions x^{T*} with (x_j^{T*}, x_h^{T*}) between (x_j^{E*}, x_h^{E*}) and $(0, d)$ do satisfy the emission constraint. This then raises the questions: Is

Figure 2.1: Set of optimal production quantities (x_j^{T*}, x_h^{T*}) when $\bar{\mu} = \mu^*$.



one of the solutions that satisfy the emission allowance better than any other solution? Is there a way to distinguish between several solutions by means of some reasonable measures? We may for example focus on the total emissions, which is obviously lowest in case $(x_j^{T*}, x_h^{T*}) = (0, d)$, but we may also consider social welfare. As the environmental regulator (or government) takes action to reduce the amount of CO_2 emitted, it generates some income. This income consists of the fixed tax collected per unit emission. We will call the total amount the regulator earns from a certain action the regulator's surplus. The regulator's surplus at a solution x^{T*} equals

$$RS^T = \sum_{k \in K} \bar{\mu} e_k x_k^{T*} = \bar{\mu}(e_j x_j^{T*} + e_h x_h^{T*}). \quad (2.16)$$

We next discuss why this way of computing the regulator's surplus helps us in distinguishing between the multiple optima. Since multiple optima means that there is a range of values for x_j^{T*} and x_h^{T*}, there is a range of values for the regulator's surplus (2.16). Recall that we would like to exclude points that are on the dotted line of Figure 2.1; that is, all points where the maximum emission allowance level would be violated. Analyzing the remaining points, one can see the trade off between maximizing the regulator's surplus and minimizing the total emissions. Then $(x_j^{T*}, x_h^{T*}) = (x_j^{E*}, x_h^{E*})$ is the point for which the regulator's surplus is maximized, whereas $(x_j^{T*}, x_h^{T*}) = (0, d)$ is the point for which the total emissions are minimized; obviously, the implications of these for "social welfare" are unclear. In addition, in reality there is no way to enforce one of these potentially preferred solutions without imposing additional conditions on the technology mixture (for example in the form of an emission allowance).

2.3.4 Characterizing Unused Technologies

As mentioned earlier, when ignoring possible equal marginal cost, any specific level of taxation, $\bar{\mu}$, induces a fixed merit order on the technologies. When a technology has the lowest total marginal cost, that is, when it is the first in the fixed merit order, it is used to satisfy (part of) the demand. This raises the following question: For any given technology, does there exist a level of fixed tax such that this technology is the first in the merit order? If the answer is "not affirmative", then there is no reason for that technology to exist with its current specifications; hence either something should be done to improve the specifications or it should be discarded.

To answer this question, we try to characterize the technologies that can not be the first in the merit order for any level of taxation. We give sufficient conditions implicating when no fixed tax level exists such that a technology is first in the merit order. We show, by means of a counter-example, that the conditions are not necessary. In addition, a technology which satisfies the sufficient conditions is not used in the optimal mixture in case of a maximum emission allowance level either.

Proposition 2.3. *Suppose that there are n firms with n different technologies, for which it holds that*

$$a_1 < a_2 < \cdots < a_n \qquad \textit{and} \qquad e_1 > e_2 > \cdots > e_n.$$

For $k \in K$, define

$$\gamma_k = \begin{cases} 0 & \textit{for } k = 1, \\ \frac{a_k - a_1}{e_1 - e_k} & \textit{for } k = 2, ..., n, \end{cases} \qquad \textit{and} \qquad \tau_k = \begin{cases} \frac{a_n - a_k}{e_k - e_n} & \textit{for } k = 1, ..., n-1 \\ \infty & \textit{for } k = n. \end{cases}$$

For any $k \in K$, if we have

$$\gamma_k > \gamma_i \qquad \textit{for at least one } i > k, \tag{2.17}$$

or

$$\tau_k < \tau_i \qquad \textit{for at least one } i < k, \tag{2.18}$$

then no $\bar{\mu}$ exists such that technology k is first in the merit order.

Proof: Let MC_k denote the effective marginal costs of technology $k \in K$, that is, $MC_k = a_k + \bar{\mu}e_k$. Then γ_k is the level of taxation at which $MC_1 = MC_k$. Furthermore, by the assumption that $e_1 > e_k$ for $k \neq 1$, we have

$$\begin{array}{ll} MC_k < MC_1 & \text{if } \bar{\mu} > \gamma_k, \\ MC_k > MC_1 & \text{if } \bar{\mu} < \gamma_k. \end{array} \tag{2.19}$$

Similarly, τ_k is the level of fixed tax at which $MC_n = MC_k$. By the assumption that $e_n < e_k$ for $k \neq n$, we get

$$\begin{array}{ll} MC_k < MC_n & \text{if } \bar{\mu} < \tau_k, \\ MC_k > MC_n & \text{if } \bar{\mu} > \tau_k. \end{array} \tag{2.20}$$

Consider any technology $k \in K$ for which (2.17) holds; that is, for some $i > k$ we have $\gamma_k > \gamma_i$. We show that for all levels of taxation a different technology, with lower marginal cost than technology k, can be found; that is, technology k will always be dominated. In particular, we claim:

$$\begin{array}{ll} \text{for } 0 \leq \bar{\mu} < \gamma_i, & MC_1 < MC_i \text{ and } MC_1 < MC_k; \\ \text{for } \gamma_i \leq \bar{\mu} < \gamma_k, & MC_i \leq MC_1 < MC_k; \\ \text{for } \gamma_k \leq \bar{\mu}, & MC_i < MC_k \leq MC_1. \end{array}$$

The first two statements follow immediately from (2.19). To see the third statement, note that we have $MC_i < MC_k$ with taxation levels between γ_i and γ_k. Since $i > k$, and hence $e_k > e_i$, an increase of $\bar{\mu}$ such that $\bar{\mu} \geq \gamma_k$ does not influence the direction of the inequality $MC_i < MC_k$.

Next, assume we have a technology $k \in K$ for which (2.18) holds; that is, for some $i < k$ we have $\tau_k < \tau_i$. We again show that for all levels of taxation another technology with lower marginal cost can be found. This time, we will have:

$$\begin{array}{ll} \text{for } \tau_i \le \bar{\mu}, & MC_n \le MC_i \text{ and } MC_n < MC_k; \\ \text{for } \tau_k \le \bar{\mu} < \tau_i, & MC_i < MC_n \le MC_k; \\ \text{for } 0 \le \bar{\mu} < \tau_k, & MC_i < MC_k < MC_n. \end{array}$$

Again, the first two statements follow immediately from (2.20). The third statement is a consequence of the assumption $e_i > e_k$. □

Next, by means of a numerical example, we show that (2.17) and (2.18) are not necessary conditions. That is, we have a situation in which none of the technologies satisfy (2.17) or (2.18); nevertheless, there is a technology for which no $\bar{\mu}$ can ensure that this technology will be the first in the merit order.

Example 2.1. Table 2.1 contains the characteristics of five technologies and the corresponding γ- and τ-values. None of the technologies satisfies (2.17) and (2.18).

Table 2.1: Characteristics of the technologies.

k	a_k	e_k	γ_k	τ_k
1	10	0.9	0	175
2	18.8	0.8	88	204
3	20.5	0.79	95.45	205.17
4	25	0.75	100	220
5	80	0.5	175	∞

Next, we compute the marginal costs of the firms for all levels of fixed tax. Table 2.2 shows that for no level of taxation technology 3 is first in the merit order. Hence, no level of fixed tax exists for which technology 3 has lowest marginal cost.

Table 2.2: Lower and upper limits of fixed tax and the corresponding technology that appears first in the merit order.

Lower limit	Upper limit	First in merit order
0	88	Technology 1
88	124	Technology 2
124	220	Technology 4
220	∞	Technology 5

Hence, the conditions mentioned in Proposition 2.3 are not necessary. We remark that it is possible to show that for less than five technologies the conditions are necessary, but with five or more technologies counter examples can be found.

Finally, we find that a technology which satisfies the sufficient conditions can not be chosen in the optimal mixture in case of a maximum emission allowance level.

Corollary 2.1. *For any maximum emission allowance level in (2.10), a technology which satisfies condition (2.17) or (2.18) is not chosen in the optimal mixture.*

Proof: Suppose that (2.17) or (2.18) hold for some given technology $k \in K$. Then, for no level of taxation technology k is first in the merit order. Hence, in the equilibrium solution to problem (2.11), the corresponding production quantity will be zero, independent of the level of taxation. Furthermore, if $\bar{\mu} = \mu^*$ is chosen, then each equilibrium solution (2.10) is a solution of (2.11). Since technology k will not be in the solution set of (2.11) for any $\bar{\mu}$, it will not be in the solution set of (2.10) for any E. □.

2.3.5 Analyzing Effects for Consumers: CO_2 Cost Pass-Through

Reducing the total amount of CO_2 emitted will obviously have a price. Since cheap and dirty technologies are replaced by costly clean technologies, total production cost and hence consumer prices increase. In this section we elaborate on to what extent additional production costs are passed through to the consumers. In particular, we consider changes in consumers' surplus as a result of regulator's actions in the form of a CO_2 emission cap or in the form of taxation.

Varian (1996) introduces consumers' surplus as follows. Each consumer is willing to pay a certain price for a good. In case of fixed, inelastic demand, the consumer is supposed to be willing to pay any price for the good. In an electricity market without demand side bidding, it is customary to assume that consumers are willing to pay any price up to $VOLL$ to obtain the electricity. Every price below $VOLL$ thus generates a surplus for the consumer. Hence, the consumers' surplus (CS) is taken as the difference between the price consumers are willing to pay and the actual price paid for each unit of demand. Given the demand d and the price of electricity p^*, the consumers' surplus can be calculated as

$$CS = (VOLL - p^*)d. \tag{2.21}$$

The total production cost (PC) can be computed by multiplying the effective marginal costs, consisting of investment, production, and emission cost, with the total production for all technologies. Hence we get

$$PC = \sum_{k=1}^{n} MC_k x_k^*. \tag{2.22}$$

We first consider the effect of going from one maximum emission allowance level to a lower maximum emission allowance level in (2.10). We distinguish between two possibilities. In the first one, both allowance levels are such that we are in case *(i)* or case *(iii)* of Proposition 2.1. In the second one, one allowance level is such that we are in case *(i)* or *(iii)* of Proposition 2.1 and the other allowance level is such that we are in case *(ii)* of Proposition 2.1, that is, the allowance level is so low that even the cleanest firm cannot satisfy demand without violating the emission constraint. We do not consider the possibility in which both allowance levels are such that we are in case (ii) of Proposition 2.1, since in this case a change of the allowance level has no effect on consumers. We show that the consumers' surplus is decreasing with the emission allowance level. We also show that in the first case the decrease in consumers' surplus equals the increase in production cost, whereas in the second case the decrease in consumers' surplus is larger than the additional production cost incurred.

We start with three different maximum allowance levels E_1, E_2, and E_3 such that $E_1 > E_2 > E_3$. Assume that E_1 and E_2 are such that we are in case *(i)* or case *(iii)* of Proposition 2.1; that is, we are in the first alternative. We know that either one or two technologies are first in the merit order and hence the market prices p^{1*} and p^{2*}, corresponding to E_1 and E_2, respectively, will be set by the effective marginal costs of these technologies. By Proposition 2.2, we know that $p^{1*} < p^{2*}$. Furthermore, the total output is d. Using (2.21) and (2.22), one can easily see that the decrease in consumers' surplus and the increase in production cost are:

$$\Delta CS = CS^{E_2} - CS^{E_1} = (VOLL - p^{2*})d - (VOLL - p^{1*})d = (p^{1*} - p^{2*})d$$

and

$$\Delta PC = PC^{E_2} - PC^{E_1} = (p^{2*} - p^{1*})d.$$

Since $p^{1*} < p^{2*}$, the consumers' surplus is decreasing as E decreases; the decrease in consumers' surplus is equal to the increase in total production cost, implying that the CO_2 cost pass-through to consumers is 100%.

Next, consider the second alternative: With the maximum allowance level E_2 we are in case *(i)* or *(iii)* of Proposition 2.1, and with the maximum allowance level E_3 we are in case *(ii)* of Proposition 2.1. That is, when the allowance level equals E_2, the consumer price and the marginal cost equal p^{2*}, and the total output equals d; when the allowance level is E_3, demand can no longer be satisfied. Then, the cleanest firm, by assumption firm n, is the only producer and is allowed to produce a quantity equal to $\frac{E_3}{e_n}$; VOLL-pricing is in effect and the price of allowances will be $\frac{VOLL-a_n}{e_n}$. Hence, the effective marginal cost of firm n equals $a_n + e_n \frac{VOLL-a_n}{e_n} = VOLL$. Using (2.21) and (2.22), the decrease in consumers' surplus and the increase in production cost are:

$$\Delta CS = CS^{E_3} - CS^{E_2} = (VOLL - VOLL)\frac{E_3}{e_n} - (VOLL - p^{2*})d =$$

$$(p^{2*} - VOLL)d$$

and

$$\Delta PC = PC^{E_3} - PC^{E_2} = VOLL\frac{E_3}{e_n} - p^{2*}d.$$

Since $\frac{E_3}{e_n} < d$ by assumption, there is a negative gap between the increase in production cost and the decrease in consumers' surplus. All additional production costs are passed through to the consumers, but there is an extra loss for the consumers due to the lower total output. Obviously, this is a situation where E_3 is set to such a low value necessitating to curb the total production.

Next, we turn our attention to the fixed tax model (2.11). We consider two different levels of fixed tax, $\bar{\mu}^1$ and $\bar{\mu}^2$ with $\bar{\mu}^1 < \bar{\mu}^2$. Suppose only one firm is producing the entire quantity demanded, d; hence, the corresponding consumer prices, p^{1*} and p^{2*}, respectively, will be set by the producing firm's effective marginal cost. Again, the corresponding decrease in consumers' surplus and increase in production cost can be found using (2.21) and (2.22), that is,

$$\Delta CS = CS^{\bar{\mu}^2} - CS^{\bar{\mu}^1} = (VOLL - p^{2*})d - (VOLL - p^{1*})d = (p^{1*} - p^{2*})d$$

and

$$\Delta PC = PC^{\bar{\mu}^2} - PC^{\bar{\mu}^1} = (p^{2*} - p^{1*})d.$$

It is easy to check that $p^{1*} < p^{2*}$. Hence, again the consumers' surplus is decreasing as $\bar{\mu}$ increases. Furthermore, the decrease in consumers' surplus is exactly equal to the increase in total production cost; hence the CO_2 cost pass-through to consumers is again 100%.

Similar to the results of Chen et al. (2008) and Bonacina and Gullí (2007), we see that under the assumption of deterministic demand and exogenous CO_2 costs (e.g. fixed tax), the CO_2 cost pass-through to consumers is 100% not only in the short run but also in a market with optimal generation capacities in the long run. In addition, we see that when CO_2 allowance prices are endogenously determined by the market, the CO_2 cost pass-through to consumers is again 100% except when the CO_2 allowance cap is too low. When the cap is too low, demand is curtailed with additional cost and the CO_2 cost pass-through to consumers exceeds 100%.

2.4 The Investment Model - Stochastic Exogenous Demand

In previous sections, we considered the impact of a CO_2 emission allowance on the technology mixture and the CO_2 cost pass through to consumers in a deterministic setting. In reality, there are uncertainties in an electricity market related to future demand, fuel prices, and emission allowances set by the regulator. Hence, extending the deterministic framework by including uncertainty provides more insight into the consequences of CO_2 regulation in reality. In this section we deal with uncertainty in demand. Realized demand is assumed to be unknown to the firms at the first stage, and will be revealed to the firms at the second stage. In particular, the first stage decisions can be seen as long term decisions, that is, capacity investments are made for a certain period, for example a year, and are based on the possible future outcomes of the second stage. The second stage decisions can then be seen as short term, for example hourly or daily, decisions. Uncertainty about the second stage outcomes may affect the choice of technology and its investment level at the first stage, that is, in order to deal with both peak and off-peak demand realizations, firms may want to invest in broader mixtures of technologies.

Whereas the deterministic setting allowed us to derive analytical results, the most convenient way to derive results in the stochastic setting is via a numerical study. When the random demand distribution is given, sampling is a handy tool for deriving numerical results. After introducing the general version of the model including stochastic demand, we will state the sampled problem. In the next section, we use the sampled problem to analyze a small network in a numerical study.

In Section 2.4.1 we briefly introduce the altered investment model including cap-and-trade and introduce how to solve this model as a large MCP using sampling. The altered fixed tax model is introduced in Section 2.4.2.

2.4.1 Introducing the Two-Stage Game Including an Emission Allowance Level

We assume that demand is determined by a random process. The demand at node $n \in N$ is denoted by $d_n(\omega)$, which has a continuous joint distribution Ψ. Here $\omega \in \Omega$ is a random vector in Ω, the space of possible outcomes. The probability distribution and its possible outcomes are known to the firms at the first stage. The realized demand will be revealed to the firms at the second stage. For each $\omega \in \Omega$ there may be a different optimal second stage outcome depending on the demand realization.

At the first stage, firms consider the expected optimal second stage profit based on the information they have on the probability distribution of demand. Hence, the objective function for firm $g \in G$ at stage one is defined as

$$\max_{x^g \geq 0} \quad E_\omega[\sum_{i \in I_g} \sum_{k \in K_g} (p_i(\omega) - c_{ik}^g - e_k \mu) y_{ik}^g(x^g, \omega)] - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_{ik}^g x_{ik}^g, \tag{2.23}$$

where optimal production quantities $y_{ik}^g(x^g, \omega)$, $g \in G$, $i \in I_g$, $k \in K_g$, and price of electricity $p_i(\omega)$, $i \in I$, for a given realization $\omega \in \Omega$, are taken from the second stage with x^g the investment quantity. μ is the price of emission allowances, that will now be determined at the first stage; that is, since a maximum emission allowance level is typically set for a certain period, for example a year, the emission allowance constraint is going to be a first stage constraint. We impose that the expected (average) emission over all realizations of the second stage should be less than or equal to the maximum allowance level E, while the price of emission allowances will have to be perpendicular to that constraint, that is,

$$0 \leq E - E_\omega \left[\sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} e_k y_{ik}^g(x^g, \omega) \right] \qquad \perp \qquad \mu \geq 0. \tag{2.24}$$

Next, we write the OPF problem that solves all second stage problems for given x and in any $\omega \in \Omega$. As a result of imposing (2.24), firms pay μ for each unit of CO_2 they emit. Hence, contrary to the OPF problem (2.7) in the deterministic case, we get a term $e_k\mu$ in the OPF's objective function. For given $x = (x^g)_{g\in G}$ and in any $\omega \in \Omega$ we solve

$$
\begin{aligned}
& Z(x,\omega) := \\
& \min_{y(\omega),f(\omega),\delta(\omega)} \sum_{g\in G}\sum_{i\in I_g}\sum_{k\in K_g}(c_{ik}^g + e_k\mu)y_{ik}^g(\omega) + VOLL \sum_{j\in N\cup I}\delta_j(\omega) \\
& \text{s.t.} \quad \begin{array}{lll}
\sum_{g\in G_j}\sum_{k\in K_g} y_{jk}^g(\omega) + \delta_j(\omega) + & & \\
f_j(\omega) \geq d_j(\omega) & (p_j(\omega)) & \forall j \in N\cup I \\
\sum_{j\in N\cup I} f_j(\omega) = 0 & (\rho(\omega)) & \\
\sum_{j\in N\cup I} PTDF_{l,j} f_j(\omega) \leq h_l & (\lambda_l^+(\omega)) & \forall l \in L \\
-\sum_{j\in N\cup I} PTDF_{l,j} f_j(\omega) \leq h_l & (\lambda_l^-(\omega)) & \forall l \in L \\
y_{ik}^g(\omega) \leq x_{ik}^g & (\beta_{ik}^g(\omega)) & \forall g\in G, i\in I_g, k\in K_g \\
y_{ik}^g(\omega) \geq 0 & & \forall g\in G, i\in I_g, k\in K_g \\
\delta_j(\omega) \geq 0 & & \forall j \in N\cup I.
\end{array}
\end{aligned}
\tag{2.25}
$$

An equilibrium to the two-stage game can be found by solving for each firm $g \in G$ the first stage problem (2.23) such that (2.24) is satisfied, while solving for each possible realization the second stage problem (2.25). As the set of possible realizations is often very large or even uncountable, we are going to use a sample of the given demand distribution as we explain in the next section.

Solving the Two-Stage Game as an MCP

In order to solve the two-stage game with random demand, we generate a random sample $\omega_1, \omega_2, ..., \omega_M$ from Ω and let $d_n(\omega_m)$ be the demand at node $n \in N$ of realization ω_m for $m \in \{1, ..., M\}$. As we have a random sample, the first stage problem (2.23) for firm $g \in G$ is approximated by

$$
\max_{x^g\geq 0} \quad \frac{1}{M}\sum_{m=1}^{M}\sum_{i\in I_g}\sum_{k\in K_g}(p_i(\omega_m) - c_{ik}^g - e_k\mu)y_{ik}^g(x^g,\omega_m) - \sum_{i\in I_g}\sum_{k\in K_g}\kappa_{ik}^g x_{ik}^g, \tag{2.26}
$$

where $p_i(\omega_m), i \in I$, and $y^g_{ik}(x^g, \omega_m), g \in G, i \in I_g, k \in K_g$, are the price of electricity and the optimal production quantities of firm g at x^g in realization ω_m, taken from the second stage. The corresponding KKT optimality condition of the sampled problem (2.26) is

$$0 \leq -\tfrac{1}{M}\sum_{m=1}^{M} \beta^{*g}_{ik}(\omega_m) + \kappa^g_{ik} \quad \perp \quad x^{*g}_{ik} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g, \tag{2.27}$$

as shown by Gürkan et al. (2013). Here, $\beta^{*g}_{ik}(\omega_m)$ is the optimal scarcity rent of firm $g \in G$ in supply node $i \in I_g$ for technology $k \in K_g$ in realization ω_m, $m \in \{1, \ldots, M\}$, taken from the second stage problem with $x = x^*$. The condition (2.27) implies that the (sample) averaged scarcity rent should cover the unit investment costs. If that is not the case, no investments will be done. The sampled market clearing condition with respect to the emission allowance is

$$0 \leq E - \frac{1}{M}\sum_{m=1}^{M}\left[\sum_{g\in G}\sum_{i\in I_g}\sum_{k\in K_g} e_k y^g_{ik}(x^{*g}, \omega_m)\right] \qquad \perp \qquad \mu^* \geq 0. \tag{2.28}$$

The interpretation of this condition is as follows. Each ω_m, $m = 1, ..., M$, can be seen as a realization occurring at day m; M is the length of a period, let's say a year. The term between the brackets is the daily emission, whereas the sum over all realizations is the yearly emission. The regulator then imposes a maximum allowance level, E, per day, that should be satisfied on average.

At the second stage, in any realization we solve the OPF problem; that is, for given $x = (x^g)_{g\in G}$, and in any realization ω_m, $m \in \{1, \ldots, M\}$, we find a solution $y^*(\omega_m)$, $\delta^*(\omega_m), p^*(\omega_m), \beta^*(\omega_m), \lambda^{*+}(\omega_m), \lambda^{*-}(\omega_m), \rho^*(\omega_m), f^*(\omega_m)$ to the following set of KKT-conditions:

$$
\begin{aligned}
0 \leq \beta_{ik}^{*g}(\omega_m) - p_i^*(\omega_m) + c_{ik}^g + & \\
e_k \mu^* &\quad \perp \quad y_{ik}^{*g}(\omega_m) \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g \\
0 \leq VOLL - p_j^*(\omega_m) &\quad \perp \quad \delta_j^*(\omega_m) \geq 0 \quad \forall j \in N \cup I \\
0 \leq \sum_{g \in G_j} \sum_{k \in K_g} y_{jk}^{*g}(\omega_m) + \delta_j^*(\omega_m) + & \\
f_j^*(\omega_m) - d_j(\omega_m) &\quad \perp \quad p_j^*(\omega_m) \geq 0 \quad \forall j \in N \cup I \\
0 \leq x_{ik}^g - y_{ik}^{*g}(\omega_m) &\quad \perp \quad \beta_{ik}^{*g}(\omega_m) \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g \\
0 \leq h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j^*(\omega_m) &\quad \perp \quad \lambda_l^{*+}(\omega_m) \geq 0 \quad \forall l \in L \\
0 \leq h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j^*(\omega_m) &\quad \perp \quad \lambda_l^{*-}(\omega_m) \geq 0 \quad \forall l \in L \\
\sum_{l \in L} PTDF_{l,j}(\lambda_l^{*+}(\omega_m) - \lambda_l^{*-}(\omega_m)) + & \\
p_j^*(\omega_m) - \rho^*(\omega_m) &= 0 \qquad \forall j \in N \cup I \\
\sum_{j \in N \cup I} f_j^*(\omega_m) &= 0.
\end{aligned}
\tag{2.29}
$$

Combining the conditions for the first stage (2.27) for all $g \in G$, the emission constraint (2.28), and the second stage conditions (2.29) for all ω_m, $m = 1, ..., M$, we get an MCP which finds an approximation of the equilibrium solution to the entire two-stage stochastic game. When the original (deterministic) problem is large (namely when we have a large network), solving this large (sampled) MCP may become too time consuming. In our numerical experiments we therefore consider a small network.

2.4.2 Introducing the Two-Stage Game Including a Fixed Tax

The fixed tax model with stochastic demand is similar to the model defined by Gürkan et al. (2013). There is no emission constraint and the price per unit emission does not depend on the demand realization. Hence, in the above sampled version of the model we replace the variable μ^* by the parameter $\bar{\mu}$ and omit the emission constraint (2.24). The resulting sampled MCP is to find a solution $x^*, y^*(\omega_m), \delta^*(\omega_m), p^*(\omega_m), \beta^*(\omega_m), \lambda^{*+}(\omega_m), \lambda^{*-}(\omega_m), \rho^*(\omega_m)$, $f^*(\omega_m)$, $m = 1, ..., M$, satisfying

$$
0 \leq -\tfrac{1}{M} \sum_{m=1}^{M} \beta_{ik}^{*g}(\omega_m) + \kappa_{ik}^g \quad \perp \quad x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g,
$$

and for each realization ω_m, $m = 1, ..., M$,

$$
\begin{array}{rcll}
0 \leq \beta_{ik}^{*g}(\omega_m) - p_i^*(\omega_m) + c_{ik}^g + & & & \\
e_k \bar{\mu} & \perp & y_{ik}^{*g}(\omega_m) \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
0 \leq VOLL - p_j^*(\omega_m) & \perp & \delta_j^*(\omega_m) \geq 0 & \forall j \in N \cup I \\
0 \leq \sum_{g \in G} \sum_{k \in K_g} y_{jk}^{*g}(\omega_m) + \delta_j^*(\omega_m) + & & & \\
f_j^*(\omega_m) - d_j(\omega_m) & \perp & p_j^*(\omega_m) \geq 0 & \forall j \in N \cup I \\
0 \leq x_{ik}^{*g} - y_{ik}^{*g}(\omega_m) & \perp & \beta_{ik}^{*g}(\omega_m) \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
0 \leq h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j^*(\omega_m) & \perp & \lambda_l^{*+}(\omega_m) \geq 0 & \forall l \in L \\
0 \leq h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j^*(\omega_m) & \perp & \lambda_l^{*-}(\omega_m) \geq 0 & \forall l \in L
\end{array}
$$

$$
\begin{array}{rll}
\sum_{l \in L} PTDF_{l,j}(\lambda_l^{*+}(\omega_m) - \lambda_l^{*-}(\omega_m)) + & & \\
p_j^*(\omega_m) - \rho^*(\omega_m) & = 0 & \forall j \in N \cup I \\
\sum_{j \in N \cup I} f_j^*(\omega_m) & = 0. &
\end{array}
$$

2.5 Numerical Study

In this section we consider a six-node example for analyzing the effect of an emission constraint and a fixed tax on the investments under demand uncertainty. In the deterministic setting in Section 2.3 we derived some results concerning the merit order and the number of technologies used at equilibrium. We show how stochastic demand results in broader technology mixtures. In addition, we investigate the adequacy of cap-and-trade and taxation when the network capacity is limited. We observe that, in order to curb CO_2 levels, investments in network capacity may be necessary. Finally, we establish a relationship between the optimal outcome of the cap-and-trade model and the taxation model. It turns out that the result partly coincides with the result derived for the deterministic setting in Section 2.3.3.

2.5.1 Experimental Data

There are three supplying firms, each located in a different node and having a unique technology at their disposal; we therefore use a single index k to distinguish between the firms. The three technologies available are coal (Coal), open cycle gas turbine (OCGT), and closed cycle gas turbine (CCGT), used by firm 1 in node 1, firm 2 in node 2, and firm 3 in node 3, respectively. In addition, there are three demand nodes, nodes 4, 5, and 6. The network, which was originally introduced by Chao and Peck (1998), is depicted in Figure 2.2. We assume infinite capacity on all transmission lines, except for lines $(1, 6)$ and $(2, 5)$, for which we assume there is a finite capacity later on. For a line (i, j) we assume that $i < j$ and transmission through l goes from i to j. Table 2.3 contains the PTDFs representing the flows through lines $(1, 6)$ and $(2, 5)$ resulting from a power injection into nodes 1 through 5; node 6 is taken as the hub node and thus has coefficients 0.

Figure 2.2: The electricity network.

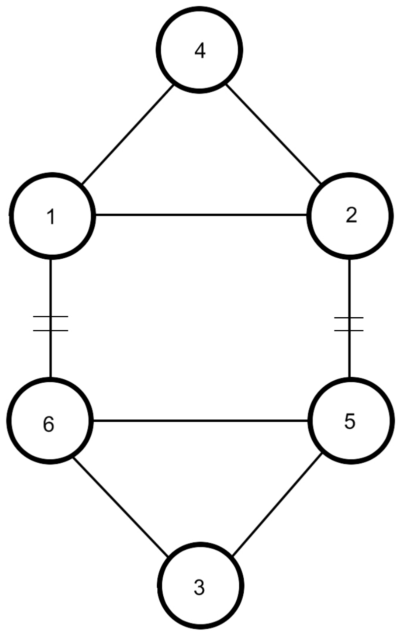


Table 2.3: Power transmission distribution factors of lines $(1, 6)$ and $(2, 5)$.

line/node	1	2	3	4	5
$(1, 5)$	0.625	0.5	0.5625	0.0625	0.125
$(2, 6)$	0.375	0.5	0.4375	-0.0625	-0.125

Table 2.4 contains the characteristics of the technologies, consisting of per unit production costs (c_k), investment costs (κ_k), both in euros per MWh, and tons of CO_2 emission (e_k).
These characteristics are taken from Ehrenmann and Smeers (2008). Notice that, in contrary to the deterministic demand case, to compute the effective

Table 2.4: Characteristics of the technologies.

	Coal	OCGT	CCGT
c_k	30	80	45
κ_k	18.3	6.8	9.1
e_k	1	0.6	0.35

marginal costs we cannot simply add the investment and production costs, since the investment quantity is not necessarily equal to the production amount in case of demand uncertainty.

Demand $d_n(\omega)$ in demand nodes $n = 4, 5, 6$ is assumed to be independently distributed. They are sampled from uniform distributions with lower bound a_n and upper bound b_n, as given in Table 2.5.

Table 2.5: Parameters for the demand.

node	a_n	b_n
4	8	12
5	3	5
6	15	20

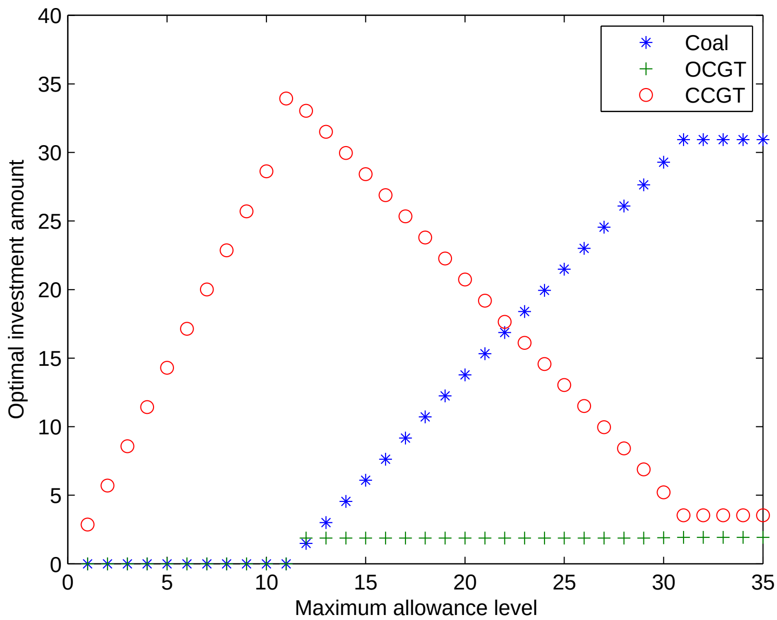
We take a sample of 3000 realizations and solve the resulting MCP using the PATH solver; see Ferris and Munson (2000). Using a 300MHz Pentium-II with 1 GB RAM, computation times are around three hours for each allowance level we consider for cap-and-trade, and 15 minutes for each level of fixed taxation. The difference in computational times is caused by the emission allowance condition (2.28) that involves production of all three firms in 3000 realizations.

2.5.2 The Effect of Maximum Allowance Level on Uncapacitated and Capacitated Networks

We consider optimal investment quantities for maximum allowance levels $E = 1, 2, ..., 35$ in three different settings, namely an uncapacitated network, transmission line $(1, 6)$ having limited capacity, and transmission line $(2, 5)$ having limited capacity.

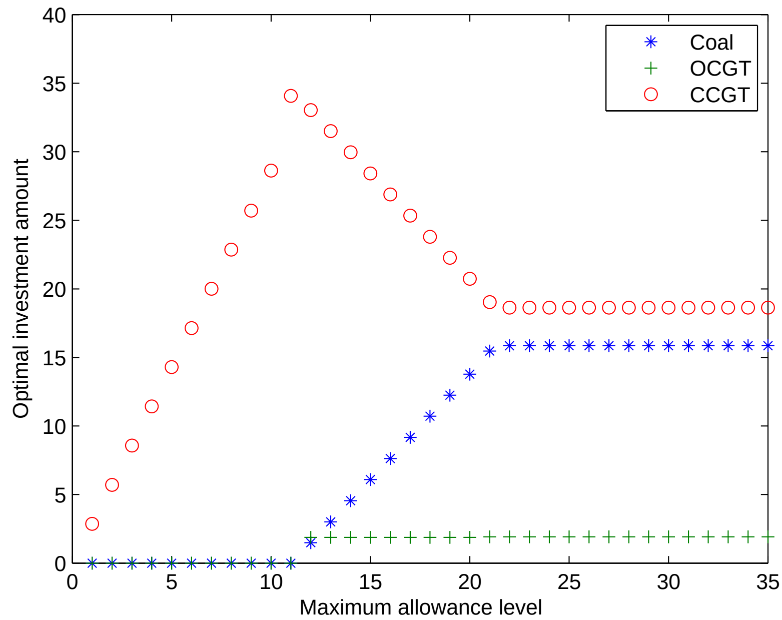
For the network without capacity constraints on the transmission lines, the optimal investment amounts in coal, OCGT, and CCGT are depicted in Figure 2.3. We observe that up to $E = 11$ only CCGT is used to satisfy the demand. Afterwards, up to $E = 31$, CCGT is gradually replaced by coal and OCGT since a more polluting mix of technologies is allowed. We observe that, in comparison to the deterministic demand case, broader mixtures are used; OCGT would never be used in the deterministic setting due to the high production cost. However, since its investment cost is low, a positive investment in OCGT turns out to be profitable in order to satisfy peak demand realizations. For $E \geq 31$ the emission constraint is not active and hence investments will be unaffected.

Figure 2.3: The optimal investment quantities for different maximum emission allowance levels in case of infinite network capacity.



Next, we consider a network with a capacity of 5 on line $(1,6)$. As a result, the PTDF-constraints will be of importance and prices and hence investment decisions will be influenced; the optimal investment amounts are depicted in Figure 2.4. Up to the level $E = 22$, Figure 2.4 and Figure 2.3 look very similar. However, starting at the point $E = 22$ higher levels of production via coal would result in exceeding the transmission capacity of the line $(1,6)$; consequently, no more CCGT is replaced by coal beyond that level. In fact the network is cleaner due to its limited transmission capacity. One would say that the network does the cleaning here, but since the allowance level is not binding this is not very interesting when it comes to CO_2 emission reduction. In addition, depending on the network structure, one may even observe more polluting mixtures in the absence of an allowance level due to limited transmission capacities.

Figure 2.4: The optimal investment quantities for different maximum emission allowance levels in case there is a transmission capacity of 5 on line $(1, 6)$.

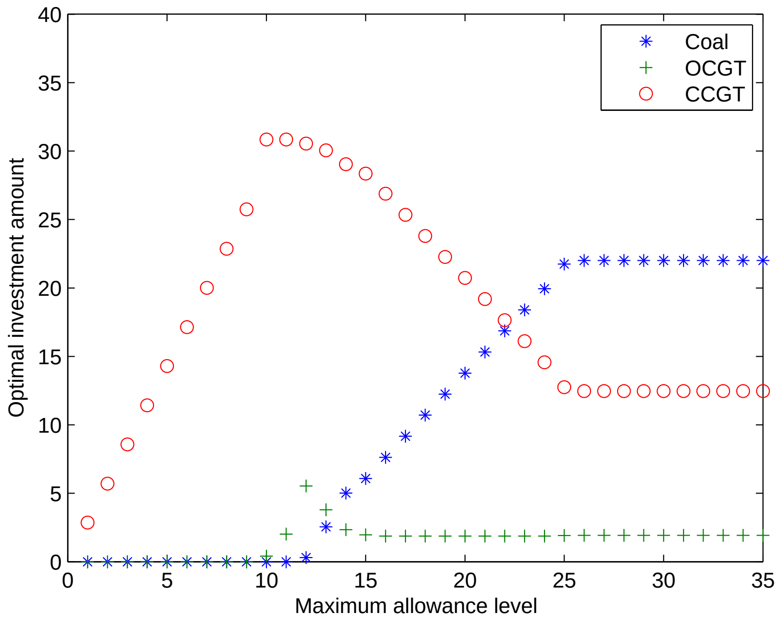


A similar behavior is observed when we choose a transmission capacity of 4 on line $(2, 5)$ and no limit on line $(1, 6)$. The optimal investment amounts are depicted in Figure 2.5. We observe two major differences between Figure 2.5 and Figure 2.3. At $E = 11$ up to $E = 14$, a strictly positive investment is made in OCGT. The transmission capacity of line $(2, 5)$ causes this behavior. The mixture of technologies chosen ensures that both the emission constraint and the transmission capacity constraint are not violated. Apparently, in this case the extra constraint associated with the transmission line capacity blocks investments in CCGT and instead motivates investments in OCGT. Clearly, this is a typical example illustrating that the shortage of transmission line capacity is preventing the cap-and-trade system to reach one of its main goals, namely to induce investments in cleaner technologies. While the regulator is creating financial incentives for firms to invest in cleaner technologies, investments in more polluting technologies are continued due to the limited network capacity. Hence, to really induce investments in cleaner technologies, investments in network capacity may also become necessary.

The other main difference between Figure 2.5 and Figure 2.3 occurs at $E = 26$. Starting at this point, higher levels of production via coal would result in exceeding the transmission capacity of line $(2, 5)$; therefore the replacement of

CCGT by coal cannot continue in the same way as observed in Figure 2.3. This actually causes a reduction in the total emissions starting at $E = 26$; in fact, a similar behavior was observed in Figure 2.4 at $E = 22$. These are examples showing that the limited network transmission capacity by itself may lower the total emissions. However, as noted before, it may also induce more investments in more polluting technologies, depending on the network structure.

Figure 2.5: The optimal investment quantities for different maximum emission allowance levels in case there is a transmission capacity of 4 on line $(2, 5)$.



2.5.3 The Effect of Taxation Level on Uncapacitated and Capacitated Networks

In this section we discuss the effect of a fixed tax on the investment quantities. In order to analyze the impact of transmission capacity on the optimal mixture of technologies and the total emissions, we again distinguish between three different settings, namely an uncapacitated network, transmission line $(1, 6)$ having limited capacity, and transmission line $(2, 5)$ having limited capacity. We take $\bar{\mu} = 0, \ldots, 20$ for the first two settings and $\bar{\mu} = 0, \ldots, 75$ for the third setting. Note that $\bar{\mu} = 0$ corresponds to an electricity market without an emission limit ($E = \infty$).

The optimal investment quantities in the network without capacities on the transmission lines are depicted in Figure 2.6. At $\bar{\mu} = 0$, the optimal investment quantities in coal, CCGT, and OCGT coincide with the optimal investment quantities in Figure 2.3 when $E \geq 34$, because the emission constraint is not binding. As $\bar{\mu}$ increases from $\bar{\mu} = 0$ to $\bar{\mu} = 8$, we observe that the investment in coal is slowly decreasing and replaced by investments in CCGT. That is caused by the increasing cost per unit production as a result of the increasing taxation. Since coal based generation emits more CO_2 per unit generation, its marginal cost increases more rapidly with $\bar{\mu}$ than the marginal cost of CCGT based generation. Therefore, as the fixed tax level increases, investments in CCGT become more attractive. Note that investments in CCGT are done although the sum of the marginal production cost, the tax paid for the emissions, and the investment cost, in other words what we have been calling the "effective" marginal cost, of CCGT may not be lowest. Recall that in the deterministic case only investments in the cheapest technology were done. In the stochastic demand case it may still be optimal to invest in a technology with higher "effective" marginal cost when the unit investment cost is relatively low. Such investments are optimal when the corresponding capacity is mostly used for peak demand realizations.

We observe that for high levels of fixed tax, from $\bar{\mu} = 9$ to $\bar{\mu} = 20$, all coal is replaced by CCGT. As we mentioned before, when the tax per unit emission increases, the "effective" marginal cost of coal increases more rapidly than the marginal cost of CCGT. Beyond $\bar{\mu} = 9$ coal becomes more expensive than CCGT, making investment in CCGT more attractive than investment in coal-based generation. Finally, we see that the investments in OCGT are at a constant level throughout to serve the peak demand.

We next consider a network with a capacity of 5 on line $(1, 6)$; the resulting optimal investments are depicted in Figure 2.7. There is one major difference between Figure 2.7 and Figure 2.6. Up to $\bar{\mu} = 8$, the investment in coal is at a much lower level, whereas the investment in CCGT is higher; this is obviously caused by the network capacity. The limited network capacity thus leads to a cleaner mixture. Beyond $\bar{\mu} = 9$, the curves look similar.

We finally put a capacity of 4 on line $(2, 5)$; the results are shown in Figure 2.8. We extend the fixed tax levels to $\bar{\mu} = 75$. This would not be interesting in the previous two cases, since results beyond $\bar{\mu} = 20$ would be the same. However, when $h_{(2,5)} = 4$, we observe different behavior. Comparing Figure 2.8 with Figure 2.6, we notice that up to $\bar{\mu} = 8$ production with coal is somewhat lower in Figure 2.8; this is again due to the limited network capacity. At $\bar{\mu} = 9$,

Figure 2.6: The optimal investment quantities for different values of fixed tax in case the network capacity is infinite.

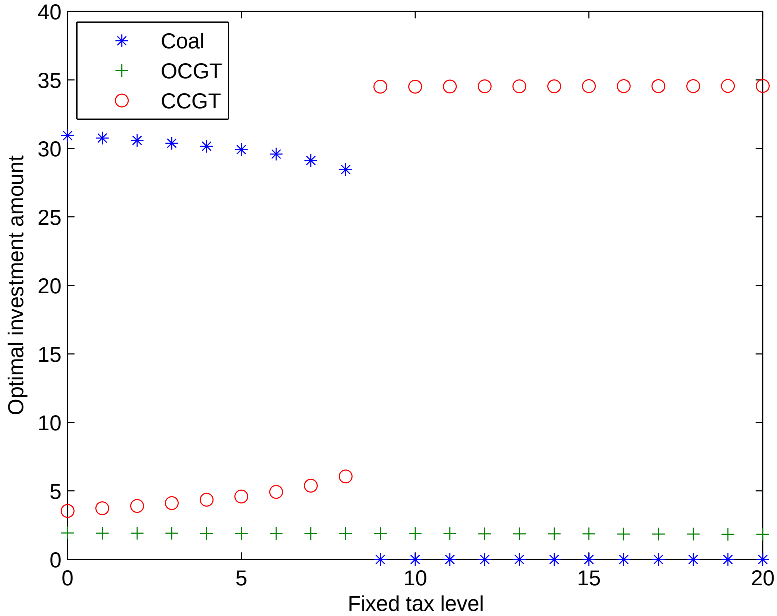


Figure 2.7: The optimal investment quantities for different values of fixed tax in case there is a transmission capacity of 5 on line $(1,6)$.

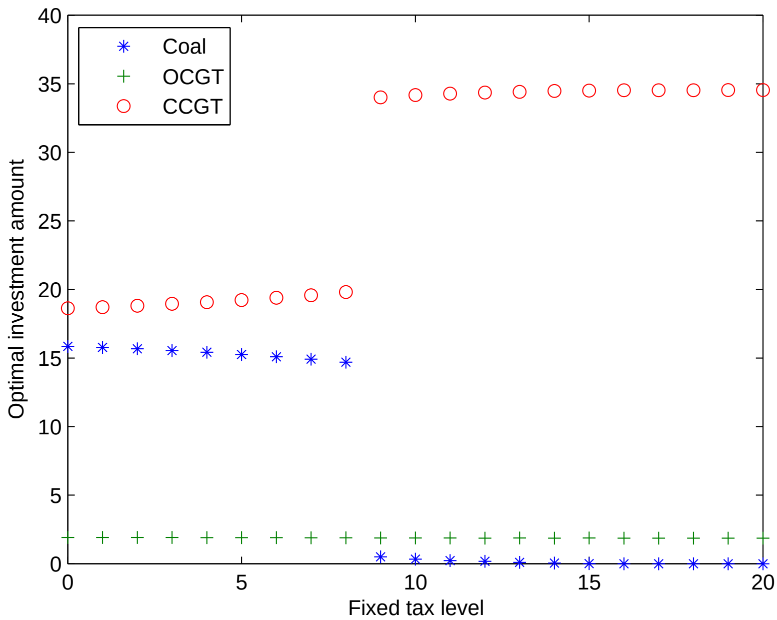
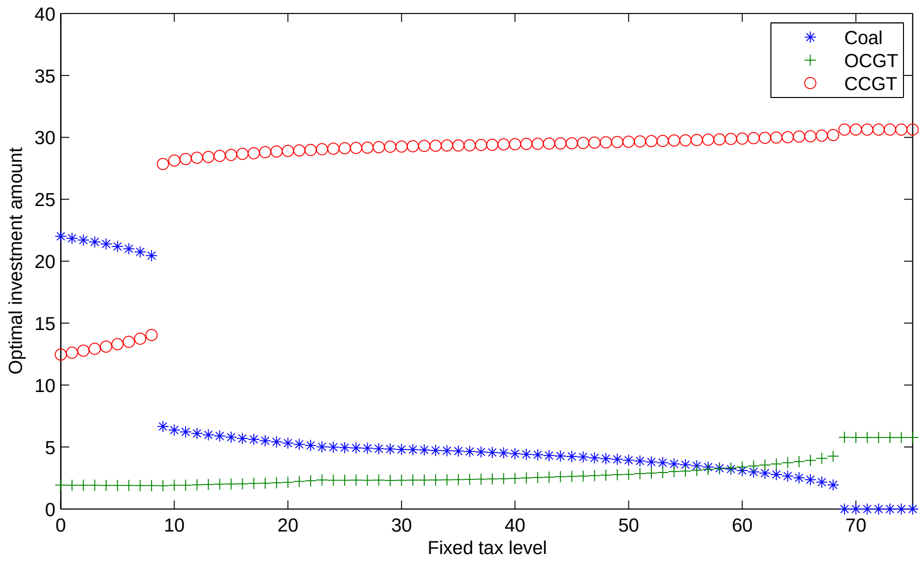


Figure 2.8: The optimal investment quantities for different values of fixed tax in case there is a transmission capacity of 4 on line $(2, 5)$.



CCGT starts dominating coal in both figures. However, in this case since the transmission capacity of the network is not sufficient to replace all coal based generation by CCGT based generation, investments in coal remain at a positive level beyond $\bar{\mu} = 9$. In other words, for high levels of carbon tax the reduction in the total emissions would be higher if line $(2, 5)$ had more capacity. This example illustrates that a financial incentive like the carbon tax may not be sufficient to curb the CO_2 levels when there is insufficient transmission capacity.

In addition, we observe that beyond $\bar{\mu} = 20$ investments in OCGT are slowly increasing and replacing investments in coal. Although coal has lower marginal production cost, the increasing taxation causes the effective marginal cost to increase up to a point where the effective marginal cost of coal and OCGT are equal; that is, at $\bar{\mu} = 69$. Starting from that point, it is less costly to invest in OCGT than in coal. We did not see such behavior before. This can be explained by the fact that in the other two examples coal was replaced by the cheaper and less polluting CCGT. Since with the current transmission capacity this is not feasible, OCGT is used. Still, since OCGT is more polluting than CCGT, the transmission capacity induces higher total emissions and hence investments in transmission capacity may be necessary to curb CO_2 levels.

2.5.4 Establishing a Relationship between Maximum Allowance Level and Taxation

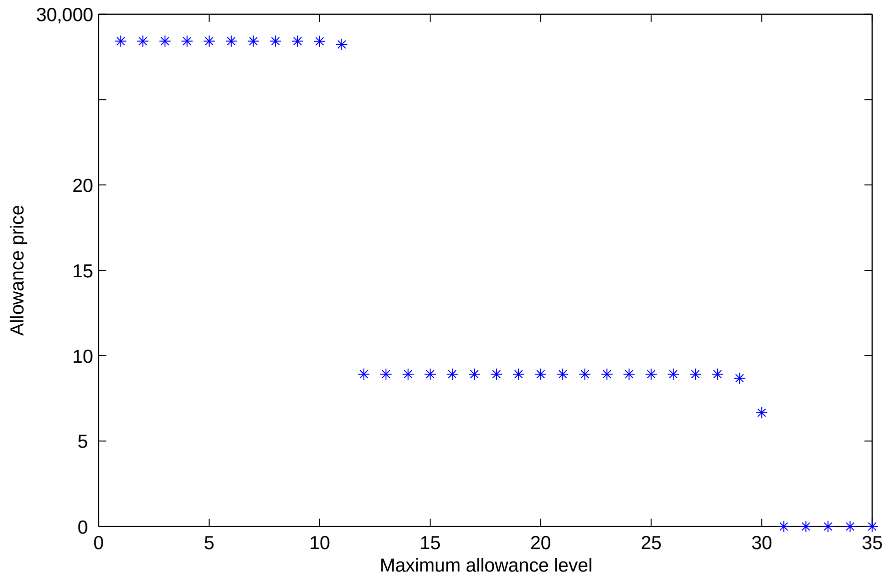
In Section 2.3.3 we considered a special case of taxation, namely the fixed tax $\bar{\mu}$ equal to the optimal price of emission allowance μ^* for some given maximum allowance level E. We found that for this taxation level multiple optima exist and that the optimal cap-and-trade solution is one of them. Between the multiple optima, a trade-off exists between minimizing pollution and maximizing regulator's surplus. A similar result can be observed when the demand is stochastic.

We assume stochastic demand and let μ^* be the optimal price of emission allowance for some given E. Next, we take $\bar{\mu} = \mu^*$ and find a solution to the taxation model. We observe two possible outcomes. Either we find a single optimum which then coincides with the optimum found in the cap-and-trade model, or, similar to what we found when demand is deterministic, we find multiple optima, of which the cap-and-trade solution is one. The latter occurs when μ^* induces two technologies with equal effective marginal cost. Our numerical results show that this is often the case. We next show an example of both possible outcomes.

In Figure 2.9 we depict the optimal allowance price for a range of E in a network with infinite capacities. We fix an E, take the corresponding optimal price of emission allowances as the fixed tax, and then compare the optimal investment quantities of both models. We first take $E = 30$. In Figure 2.9 we observe that the corresponding price of emission allowances is $\mu^* = 6.66$. Taking $\bar{\mu} = 6.66$ results in a single optimum, see Figure 2.6. This optimum coincides with the optimal investment quantities in the cap-and-trade model when $E = 30$, as can be seen in Figure 2.3.

Next we consider $E = 15$. In Figure 2.9 we observe that the corresponding price of emission allowances is $\mu^* = 8.92$. Taking $\bar{\mu} = 8.92$ will make the effective marginal cost of coal and CCGT equal. As a result a central decision maker would be indifferent between the two technologies. When solving the taxation model with this particular taxation level, we find a range of optimal solutions. This can be seen in Figure 2.6, where at a certain point a jump occurs. This jump occurs exactly at $\bar{\mu} = 8.92$; all points in between represent optimal investment quantities. We thus have multiple optima, of which some may violate the allowance level of 15 and some may be cleaner. Similar to the deterministic case, as discussed in Section 2.3.3, there exists a trade-off between minimizing pollution and maximizing regulator's surplus. One of

Figure 2.9: The optimal price of emission allowance for different maximum emission allowance levels in case of infinite network capacity.



the multiple solutions results in a total emission of exactly 15. That solution coincides with the cap-and-trade solution.

Concluding, taking the optimal price of emission allowances as the fixed tax either results in the same unique optimal solution, or results in multiple optima of which the cap-and-trade solution is one.

2.6 Conclusions

In this chapter we address the effect of two possible actions at the disposal of a regulator to curb CO_2 emission levels and to give power generating firms incentives to invest in cleaner technologies. In a stylized version of the investment model with no network effects and deterministic inelastic demand, we show that it is optimal to use a mixture of a relatively clean and a relatively dirty technology to satisfy the demand under the cap-and-trade system. For a fixed ceiling on the total emissions and for different demand levels, there can be a different optimal mixture of technologies. We also propose an algorithm that finds such an optimal mixture. Furthermore, we analytically show that the price of electricity and the price of allowances increase as the ceiling on the total emissions decreases; and the extra production costs incurred are fully passed through to the consumers.

In comparison, when a fixed carbon tax per unit emission is charged, we observe a fixed merit order on the firms. We give a characterization of technologies for which no fixed tax level exists, such that they are first in the merit order. Consequently, these technologies will never be used in the optimal technology mixture. We show that these technologies will not be in the optimal mixture in case of a cap-and-trade system either.

We also analyze the investment model with network effects and stochastic inelastic demand through a numerical study and discuss the implications of limited network capacity. We find that due to demand uncertainty a broader mix of technologies is used in the optimal mixture, both with cap-and-trade and carbon tax. We observe that in case of cap-and-trade, limited network capacity may cause that investments in dirty technologies are necessary to satisfy the demand without violating the transmission constraints. Hence, cleaner mixtures of technologies are not necessarily induced when there is limited network capacity. In case a carbon tax per unit emission is charged, we observe that limited transmission capacity puts a limit on the replacement of dirty technology by clean technology. In other words, the reduction of the total emissions due to taxation would be higher if there was more available transmission capacity. Hence, in order to curb CO_2 levels, investments in network capacity may be necessary. Finally, we establish a connection between the equilibria in both models and find that, when taking the optimal price of emission allowances as the fixed tax, multiple optima may exist. When this is the case, some optima violate the emission allowance constraint, and one of the optima coincides with the cap-and-trade solution.

2.7 Appendix: Proof of the Forward Algorithm

In this appendix we will show the following for the Forward Algorithm:

1. Given a j^*, choosing $h^* \in \bar{H}$ which minimizes the given quotient in Step 2 guarantees that (2.12) is satisfied.

2. Given a j_E, if we find a corresponding $h_E \notin H$ in Step 2, then we need to update $j^* = h_E$. Else, for at least one firm, (2.12) is violated.

Proof of 1. Suppose $J \neq \emptyset$ and define $\bar{J} = \{1\}$ and $\bar{H} = \{2, ..., n\}$. In Step 2 of the algorithm we choose

$$h^* = \arg\min_{h \in \bar{H}} \{\frac{a_h - a_1}{e_1 - e_h}\}.$$

For the sake of clarity, assume throughout the proof that the minimum found in this step is unique. If not, one can still find an optimal solution as we argue in the *Observation* below. By the choice of h^* we obtain for every $k \in \bar{H}$,

$$0 \leq \frac{a_k - a_1}{e_1 - e_k} - \frac{a_{h^*} - a_1}{e_1 - e_{h^*}} = \frac{a_k(e_1 - e_{h^*}) + a_1(e_{h^*} - e_k) + a_{h^*}(e_k - e_1)}{(e_1 - e_k)(e_1 - e_{h^*})}.$$

Since the denominator is positive, (2.12) follows. Note that equality holds for $k = h^*$.

Suppose we have $h^* \notin H$. Hence we define new candidate sets $\bar{J} = \{1, ..., h^*\}$ and $\bar{H} = \{h^* + 1, ..., n\}$. We take $j^* = h^*$ and find the new h^*, denoted by h^{**}, as

$$h^{**} = \arg\min_{h \in \bar{H}} \{\frac{a_h - a_{h^*}}{e_{h^*} - e_h}\}.$$

Next, we prove that by this choice (2.12) is satisfied for all k. Notice that, since we altered j^* and h^*, (2.12) will have the following form:

$$a_k(e_{h*} - e_{h**}) + a_{h*}(e_{h**} - e_k) + a_{h**}(e_k - e_{h*}) \geq 0 \quad \forall k \in \{1, ..., n\}. \quad (2.30)$$

We will show that (2.30) holds in two parts; first for $k \in \bar{H}$, then for $k \notin \bar{H}$.

For every $k \in \bar{H}$

$$0 \leq \frac{a_k - a_{h^*}}{e_{h^*} - e_k} - \frac{a_{h^{**}} - a_{h^*}}{e_{h^*} - e_{h^{**}}} = \frac{a_k(e_{h^*} - e_{h^{**}}) + a_{h^*}(e_{h^{**}} - e_k) + a_{h^{**}}(e_k - e_{h^*})}{(e_{h^*} - e_k)(e_{h^*} - e_{h^{**}})},$$

and (2.30) follows for $k \in \bar{H}$, since the denominator is positive.

For every $k \notin \bar{H}$, first observe that (2.12) holds for every k when $j = 1$ and $h = h^*$, as found in the first iteration. That is,

$$a_k(e_1 - e_{h^*}) + a_1(e_{h^*} - e_k) + a_{h^*}(e_k - e_1) \geq 0 \quad \forall k \in \{1, ..., n\}. \tag{2.31}$$

In particular, for $k = h^{**}$, we have

$$a_{h^{**}}(e_1 - e_{h^*}) + a_1(e_{h^*} - e_{h^{**}}) + a_{h^*}(e_{h^{**}} - e_1) \geq 0. \tag{2.32}$$

Note that (2.31) implies

$$a_k \geq \frac{a_1(e_k - e_{h^*}) + a_{h^*}(e_1 - e_k)}{(e_1 - e_{h^*})} \quad \forall k \notin \bar{H}. \tag{2.33}$$

We will finally use this together with (2.32) to show that (2.30) holds for $k \notin \bar{H}$:

$$a_k(e_{h^*} - e_{h^{**}}) + a_{h^*}(e_{h^{**}} - e_k) + a_{h^{**}}(e_k - e_{h^*}) \geq$$

$$\frac{a_1(e_k - e_{h^*}) + a_{h^*}(e_1 - e_k)}{(e_1 - e_{h^*})}(e_{h^*} - e_{h^{**}}) + a_{h^*}(e_{h^{**}} - e_k) + a_{h^{**}}(e_k - e_{h^*}) =$$

$$\frac{e_k - e_{h^*}}{e_1 - e_{h^*}}\left(a_{h^{**}}(e_1 - e_{h^*}) + a_1(e_{h^*} - e_{h^{**}}) + a_{h^*}(e_{h^{**}} - e_1)\right) \geq 0.$$

The first inequality follows from (2.33) and $e_{h^*} > e_{h^{**}}$, and the last inequality follows from (2.32) and by the fact that $e_k \geq e_{h^*}$ $\forall k \notin \bar{H}$. This shows that (2.30) holds for every k. □

Proof of 2. Given a j^*, say j_E, suppose we found a corresponding h_E with $h_E \notin H$. Then, we define the new candidate sets $\bar{J} = \{1, ..., h_E\}$ and $\bar{H} = \{h_E + 1, ..., n\}$. Suppose in the next iteration we do not choose $j^* = h_E$, but $j^* = k_0$ for some $k_0 \in \{j_E + 1, ..., h_E - 1\}$ and find the corresponding new h^* as $m_0 \in \bar{H}$ using Step 2. We show that with this choice (2.12) will be violated for $k = h_E$, that is

$$a_{h_E}(e_{k_0} - e_{m_0}) + a_{k_0}(e_{m_0} - e_{h_E}) + a_{m_0}(e_{h_E} - e_{k_0}) \leq 0. \tag{2.34}$$

First observe that, since for j_E we found h_E, (2.12) holds for every k. In particular, for $k = k_0$ and $k = m_0$; that is,

$$a_{k_0}(e_{j_E} - e_{h_E}) + a_{j_E}(e_{h_E} - e_{k_0}) + a_{h_E}(e_{k_0} - e_{j_E}) \geq 0 \tag{2.35}$$

and

$$a_{m_0}(e_{j_E} - e_{h_E}) + a_{j_E}(e_{h_E} - e_{m_0}) + a_{h_E}(e_{m_0} - e_{j_E}) \geq 0. \tag{2.36}$$

Furthermore, (2.35) implies

$$a_{k_0} \geq \frac{a_{j_E}(e_{k_0} - e_{h_E}) + a_{h_E}(e_{j_E} - e_{k_0})}{(e_{j_E} - e_{h_E})}. \tag{2.37}$$

We finally use this together with (2.36) to show (2.34):

$$a_{h_E}(e_{k_0} - e_{m_0}) + a_{k_0}(e_{m_0} - e_{h_E}) + a_{m_0}(e_{h_E} - e_{k_0}) \leq$$

$$a_{h_E}(e_{k_0} - e_{m_0}) + \frac{a_{j_E}(e_{k_0} - e_{h_E}) + a_{h_E}(e_{j_E} - e_{k_0})}{(e_{j_E} - e_{h_E})}(e_{m_0} - e_{h_E}) + a_{m_0}(e_{h_E} - e_{k_0}) =$$

$$\frac{e_{h_E} - e_{k_0}}{e_{j_E} - e_{h_E}} \left(a_{m_0}(e_{j_E} - e_{h_E}) + a_{h_E}(e_{m_0} - e_{j_E}) + a_{j_E}(e_{h_E} - e_{m_0})\right) \leq 0.$$

The first inequality follows from (2.37) and the fact that $e_{m_0} < e_{h_E}$, and the last inequality follows from (2.36) and the fact that $e_{h_E} < e_{k_0} < e_{j_E}$. This shows that (2.34) holds. Hence, (2.12) is violated for $k = h_E$. □

Observation: As a final remark, notice that when taking the argmin in Step 2 of the algorithm, the minimum may not be unique. Suppose that for a j^* we find that both h_1^* and h_2^*, with $h_1^* < h_2^*$, satisfy

$$h_i^* = \arg\min_{h \in \bar{H}} \{\frac{a_h - a_{j^*}}{e_{j^*} - e_h}\}, \quad i = 1, 2.$$

Next we show that

$$\frac{a_{h_1^*} - a_{j^*}}{e_{j^*} - e_{h_1^*}} = \frac{a_{h_2^*} - a_{j^*}}{e_{j^*} - e_{h_2^*}} = \frac{a_{h_2^*} - a_{h_1^*}}{e_{h_1^*} - e_{h_2^*}}. \tag{2.38}$$

Notice that these quantities are actually the resulting prices of emission allowances, μ^*, if we would choose one of the pairs in $\{j^*, h_1^*, h_2^*\}$ in the optimal mixture. This means that for all pairs in $\{j^*, h_1^*, h_2^*\}$, μ^* is equal and as a consequence p^* is equal as well. Hence, if for one pair (2.12) is satisfied, it is automatically satisfied for the other pairs in case the argmin finds more than one minimizer. The first equality in (2.38) follows immediately since both h_1^* and h_2^* give a minimum. Rewriting the first equality gives

$$e_{j^*} - e_{h_1^*} = \frac{a_{h_1^*} - a_{j^*}}{a_{h_2^*} - a_{j^*}}(e_{j^*} - e_{h_2^*}). \tag{2.39}$$

The second equality in (2.38) is derived as follows:

$$\frac{a_{h_1^*} - a_{j^*}}{e_{j^*} - e_{h_1^*}} - \frac{a_{h_2^*} - a_{h_1^*}}{e_{h_1^*} - e_{h_2^*}} = \frac{a_{h_1^*}(e_{j^*} - e_{h_2^*}) + a_{j^*}(e_{h_2^*} - e_{h_1^*}) + a_{h_2^*}(e_{h_1^*} - e_{j^*})}{(e_{j^*} - e_{h_1^*})(e_{h_1^*} - e_{h_2^*})} =$$

$$\frac{a_{h_1^*}(e_{j^*} - e_{h_2^*}) + a_{j^*}(e_{h_2^*} - e_{j^*}) + (a_{h_2^*} - a_{j^*})(e_{h_1^*} - e_{j^*})}{(e_{j^*} - e_{h_1^*})(e_{h_1^*} - e_{h_2^*})} =$$

$$\frac{a_{h_1^*}(e_{j^*} - e_{h_2^*}) + a_{j^*}(e_{h_2^*} - e_{j^*}) + (a_{h_1^*} - a_{j^*})(e_{h_2^*} - e_{j^*})}{(e_{j^*} - e_{h_1^*})(e_{h_1^*} - e_{h_2^*})} = 0.$$

The first and second equality follow by rewriting, whereas the third equality follows by replacing $(e_{h_1^*} - e_{j^*})$ according to (2.39). Hence, for all pairs the fraction is equal and any pair can be chosen as an optimal pair.

CHAPTER 3

MODELING AND ANALYSIS OF RENEWABLE ENERGY OBLIGATIONS AND TECHNOLOGY BANDINGS IN THE UK ELECTRICITY MARKET

3.1 Introduction

In decentralized electricity markets, firms are mainly focussed on maximizing their profits while competing with other firms. Investments in cheap and often polluting technologies tend to serve these goals well. This is in conflict with the goals set by governments as they aim at reducing pollution and want therefore to create incentives to make investments in cleaner technologies more attractive. Charging firms for their carbon dioxide (CO_2) emissions through either taxation or a cap-and-trade system in which firms need to buy emission permits, are two possible actions regulators can implement to make production with polluting technologies more expensive and therefore financially less attractive. Instead of charging firms for their emissions, governments may also hand out subsidies to firms for producing with clean technologies. That way investments in renewable technologies, which usually come with very high investment costs, become profitable. One can make a distinction between direct subsidies (usually in the form of Feed-in Tariffs) and indirect subsidies. In this chapter we discuss the latter in more detail.

An indirect subsidy is usually given in the form of a renewable obligation. In several US states and in European countries like Belgium, Poland, Romania, Sweden, Italy, and UK, a renewable obligation is in effect.[1] The renewable obligation is a target on the proportion of electricity that should come from renewable resources and is set by the regulator on one group of operators in the market. Usually the obligation is imposed on consumption, through electricity sellers. In Italy however, the obligation is on the generators. So-called green certificates are used to show compliance to the target, and typically one such certificate represents 1 MWh of renewable electricity production. At the end of each obligation period, often a year, each seller or producer should submit a certain number of certificates to the regulator. When not satisfying the target, typically a buy-out fine has to be paid. The latter comes with an opportunity cost that puts a value on each certificate, which forms the price a seller is willing to pay to a renewable generator. The reward that generators receive adds to the short-term profits in a way that high long-term investment costs can be covered. Certificates can also be traded on a secondary market and as a consequence the renewable obligation does not oblige individual generators to produce a certain part of their electricity generation with renewable resources.

The UK and Italy form an exception to the system where one certificate represents 1 MWh of renewable electricity. In these countries certificates are banded according to technology, meaning that for different (renewable) technologies a different number of certificates is handed out per MWh of production. These so-called banding systems in the UK and Italy can help in encouraging investments in less developed technologies as to make them more competitive in the long run. This way it can overcome one of the shortcomings of the regular renewable obligation which is known to single out the most developed technologies, namely onshore wind power and to a lesser extent landfill gas (see Meyer (2003), Wood and Dow (2011), and Verbruggen and Lauber (2012)). Although these technologies may be financially attractive, due to all kind of geographical constraints and opposition against onshore wind farms it is unlikely that the renewable obligation target can be met in the long run without investments in other renewable technologies like offshore wind, as emphasized by Toke (2011) and Wood and Dow (2011).

In this chapter we investigate the effects of imposing a renewable obligation and introducing tradable green certificates. In particular, we focus on the (original) obligation system in the UK and take a closer look at their banding system in a mathematical and analytical framework. In order to analyze the

[1] For overviews of different support mechanisms across Europe, see Fouquet and Johansson (2008) and Koster et al. (2011).

system we extend the two stage investment model as introduced in Gürkan et al. (2013) by incorporating renewable obligations. In the mathematical model, investments are considered long-term decisions (for example yearly) which take place at the first stage. Production, transmission, and market clearing are short-term decisions (for example hourly or daily) that take place in the spot market, referred to as the second stage, which can be repeated several times. We assume perfect competition at both stages meaning that firms are price takers.

In order to make our model a good representation of reality and to keep results analytically tractable, we will make two simplifying assumptions with respect to the UK system. First of all, as mentioned earlier, in the UK the renewable obligation is imposed on total electricity sales. We assume though, that the firms producing power are selling their power directly to consumers, meaning that the obligation will be on electricity production. The second simplifying assumption concerns the trading of certificates. Recall that in the original obligation system each MWh of renewable energy production is rewarded with one renewable certificate. At the end of each period (typically a year) generators should submit a certain number of certificates proportional to the total production to show the obligation is met. In reality, the certificates can be traded daily on a secondary market and will have a certain value determined by the short term demand for certificates. Trading would be done on a daily basis and result in day to day variations in the value of a certificate. We overlook the micro details of the secondary trading market and therefore ignore the daily trading possibilities. Instead, we consider the average certificate value that holds over the year, which is directly related to the yearly obligation target. This average value is the reward the regulator pays firms per certificate. At the same time the consumer price is regulated in the form of a mark-up, as to cover the certificate payments.

After modeling the original renewable obligation into a mathematical framework, we take a closer look at the banding system that was introduced in the UK in 2009.[2] Under the banding policy, production with different renewable technologies is rewarded with a different number of certificates. As a consequence, some model adjustments need to be done; that is, we replace the obligation on production by one on certificates, and modify the pricing scheme defining the prices paid for production with non-renewable and renewable production and the consumer price. We analyze the consequences of

[2]For a description of the UK (banding) system, see Constable and Barfoot (2008) and Clark (2008).

the new policy and argue that the system can be effective in giving incentives to invest in less established technologies, but as an undesirable side effect may result in a more polluting mixture of technologies.

As a potential remedy for this negative side effect, we propose an alternative banding policy. In this alternative, production with different renewable technologies will be rewarded with a different amount of certificates in the same way as in the UK banding system. However, the obligation will be on renewable production rather than on certificates, similar to how it is done in the Italian banding system.[3] Different from the Italian banding system, in which the regulator buys excess certificates, we make a modification to the reward per certificate in order to guarantee that rewards paid to firms are covered by mark ups paid by consumers, that is, to guarantee revenue adequacy for the regulator.

We then analyze the original obligation system, the UK banding system, and our proposed alternative in a numerical study where we assume uncertain demand and uncertain generation output of renewable resources. We focus on a small network with two non-renewable technologies (coal and combined cycle gas turbine (CCGT)) and three renewable technologies (onshore wind (ONW), offshore wind (OFFW), and landfill gas (LFG)) and obtain investment quantities, prices, and CO_2 emissions in all three systems. A key observation in all three systems is that the higher the obligation target, the more coal is replaced not only by renewable technologies, but also by CCGT which acts as a peak technology in case of high demand and/or low wind output. That way CO_2 emissions are curbed both by the increased renewable capacity and by the replacement of coal by the cleaner CCGT. Introducing the UK banding system has the effect of giving incentives for investments in OFFW, which were not present in the original system. However, bandings fail to create the right incentives when the obligation target is set too low. In that case the UK banding system may result in a cleaner mixture (but without OFFW) and overshoot the original target on production. On the other hand, when there are incentives to invest in OFFW we observe that for high obligation targets on certificates, the original target on production will not be satisfied. Hence, the UK banding system may lead to a more polluting technology mixture. Furthermore we observe that consumer prices in the UK banding system are not affected by having the more expensive OFFW in the mixture. Comparing the UK system to our alternative, we observe that the alternative needs higher obligation targets in order to create incentives for OFFW investments.

[3] For a description of the Italian system, see Giovannetti (2009).

When the obligation target is high enough for OFFW to be in the optimal mixture, we observe that consumer prices increase and exceed those obtained in the other systems.

We also analyze the effect of a decrease in the investment cost of OFFW. We find that investments are very sensitive to such a cost reduction and that under both banding policies there will be more OFFW in the system. Interestingly, although the investment quantities change, consumer prices in the UK banding system and CO_2 emissions in the alternative banding system remain unchanged. On the other hand, we observe a significant increase in consumer prices in the alternative system and considerably increased levels of CO_2 emissions in the UK banding system.

To summarize, the key findings in this chapter are: First, the UK banding system may result in higher levels of CO_2 emissions compared to the original obligation system when OFFW is in the technology mixture; the alternative banding system proposes a possible solution for this undesirable side effect, albeit with relatively higher and less stable consumer prices. Second, cost reductions in a technology with high bandings (that is, OFFW), lead to increased levels of CO_2 emissions in the UK banding system and increased consumer prices in the alternative system. These are obviously negative side effects of banding systems, implying that as costs reduce, financial support and hence bandings should be reduced accordingly.

This chapter is organized as follows. We introduce the basic electricity market investment model in Section 3.2 and expand this model to include the renewable obligation in Section 3.3. In Section 3.4 we introduce the concept of bandings. We explain the details of the UK banding system and propose an alternative system. In Section 3.5 we introduce uncertainty into the models. Both the generation output of renewable resources and electricity demand will be uncertain. The numerical study is carried out in Section 3.6. Section 3.7 concludes.

3.2 The Electricity Market Investment Model

In this section we introduce the electricity market investment model. Given is an electricity grid with supply nodes at which firms owning generation plants produce electricity using their technologies that are renewable or non-renewable, and demand nodes at which consumers with, by assumption, inelastic demand are located. Supply and demand nodes are connected by means of transmission lines with a given capacity, forming a network. Typically, when power is transmitted from one node to another through a transmission line, flows in the entire network are affected; furthermore, electricity cannot be stored. Firms at supply nodes make decisions in two stages. At the first stage all firms simultaneously maximize their profits while determining their optimal production capacity. Their profits are dependent on the equilibrium outcome at the second stage, which in return is dependent on the investment decisions of all firms at the first stage. The equilibrium at the second stage is between firms maximizing their profits while producing electricity given their production capacity in the first stage, a transmission system operator (TSO) owning the transmission grid, who is maximizing its own profits and taking care of the flows between supply and demand nodes, and subject to two types of market clearing conditions, an imposed price cap when there is unsatisfied demand, and a condition guaranteeing that demand is satisfied in all nodes. We assume perfect competition, and hence at both stages firms are not aware of the fact that they can influence prices. Since at the market equilibrium investment decisions of firms (indirectly) depend on decisions of other firms, we have a two stage game between the firms. When the first stage is solved to optimality and the second stage is at equilibrium, for none of the entities it is profitable to deviate and thus a perfectly competitive equilibrium is obtained.

A suitable mathematical framework for the two-stage game is presented in Gürkan et al. (2013). They model the electricity market in a two-stage setting and shows that under perfect competition the resulting two-stage game can be written as a single optimization problem when the model is deterministic, or as a standard two stage stochastic program in case of (demand) uncertainty at the second stage. In Chapter 2, the same model is used for analyzing the consequences of imposing a taxation per unit emission or a cap-and-trade system. Instead of charging firms for their emissions, in this chapter we analyze the effects of imposing an indirect subsidy in the form of a renewable obligation, which will be the subject of Section 3.3. Before imposing the obligation, we provide an overview of the electricity market investment model as formulated in Gürkan et al. (2013).

We make two adjustments to the models in Gürkan et al. (2013) and Chapter 2. First of all, for simplicity we assume that each firm has its own technology and is operational at all supply nodes (but not necessarily producing) for notational convenience. Second, we explicitly distinguish between a set of non-renewable technologies and a set of renewable technologies. The sets, parameters, and variables are given below.

Sets:

N	:	the set of demand nodes
I	:	the set of supply nodes
K^N	:	the set of non-renewable technologies
K^R	:	the set of renewable technologies
K	:	the set of all technologies ($K := K^N \cup K^R$)
L	:	the set of electricity transmission lines connecting nodes in the network.

Parameters:

c_{ik}	:	unit production cost at supply node $i \in I$ for technology $k \in K$
κ_{ik}	:	unit investment cost at supply node $i \in I$ for technology $k \in K$
d_n	:	demand at demand node $n \in N$
M_k^R	:	ceiling on investments in renewable technology $k \in K^R$
$PTDF_{l,j}$	:	power transmitted through line $l \in L$ due to one unit of power injection into node $j \in N \cup I$
h_l	:	capacity limit of line $l \in L$
$VOLL$	:	value of unserved energy or lost load.

Variables:

x_{ik}	:	generation capacity investment in technology $k \in K$ at supply node $i \in I$
y_{ik}	:	quantity of power generated at supply node $i \in I$ by using technology $k \in K$
f_j	:	net power flow dispatched by the TSO to node $j \in N \cup I$
δ_j	:	unserved demand at node $j \in N \cup I$
p_j^c	:	electricity price at node $j \in N \cup I$
p_{ik}^N	:	price non-renewable technology $k \in K^N$ at supply node $i \in I$ gets per unit sold
p_{ik}^R	:	price renewable technology $k \in K^R$ at supply node $i \in I$ gets per unit sold.

In the remainder of the chapter, variables may get superscripts N and R depending on their corresponding technology in K^N and K^R, respectively. In addition, for $k \in K$ we define $x_k = (x_{ik})_{i \in I}$ and $y_k = (y_{ik})_{i \in I}$, the vectors containing investments and production, respectively, of technology $k \in K$ in all supply nodes.

We next formulate the first and second stage problems. At the first stage each firm $k \in K$ determines its optimal investment quantities x_k in all supply nodes, in order to maximize the optimal second stage profit minus the investment cost. The optimal second stage profit of technology $k \in K$ at supply node $i \in I$ is the price p_{ik} minus production cost c_{ik}, times the optimal second stage production y_{ik} as a function of investments x_k. Unit investment costs for technology $k \in K$ in supply node $i \in I$ are κ_{ik}. The first stage objective function for a non-renewable generator $k \in K^N$ looks as follows:

$$\max_{x_k^N \geq 0} \sum_{i \in I} (p_{ik}^N - c_{ik}^N) y_{ik}^N(x_k^N) - \sum_{i \in I} \kappa_{ik}^N x_{ik}^N. \tag{3.1}$$

$y_{ik}^N(x_k^N)$ is the optimal production quantity of non-renewable technology $k \in K^N$ at supply node $i \in I$ at the second stage if x_k^N are the investment quantities. Since firms are assumed to be price takers, for $i \in I$, p_{ik}^N is taken as a parameter. Without price regulation, p_{ik}^N, $i \in I$, $k \in K^N$, is typically equal to the electricity price in node i, p_i^c, but depending on the pricing scheme p_{ik}^N may be altered when for example an additional fee for a certificate or permit has to be paid. For a renewable generator $k \in K^R$ the first stage problem looks similar, namely:

$$\max_{x_k^R \geq 0} \sum_{i \in I} (p_{ik}^R - c_{ik}^R) y_{ik}^R(x_k^R) - \sum_{i \in I} \kappa_{ik}^R x_{ik}^R. \tag{3.2}$$

Again, $y_{ik}^R(x_k^R)$ is the optimal production quantity of renewable technology $k \in K^R$ at supply node $i \in I$ at the second stage if x_k^R are the investment quantities. The price p_{ik}^R is taken as a parameter. Without any price policies imposed by the regulator, typically in each node $i \in I$ and for any $k \in K^R$, the price p_{ik}^R is equal to the electricity price p_i^c, but depending on the pricing scheme imposed it may be altered. As we will see in the next section, introducing a renewable obligation may result in a difference between p^N and p^R.

In addition, there can be a ceiling on capacity investments due to for example regulation or physical limitations. This is typical for some renewable technologies like landfill gas. Hence, we add the following constraint that should be satisfied for technology $k \in K^R$:

$$\sum_{i \in I} x_{ik}^R \leq M_k^R \qquad (\zeta_k^R), \tag{3.3}$$

where ζ_k^R is the nonnegative dual price associated with the ceiling. We provide an interpretation of this dual price at the end of this section.

At the second stage each firm determines its optimal production quantities while maximizing its second stage profit, subject to the capacity constraint.

The investment quantities from the first stage are given and treated as parameters. For each renewable technology $k \in K^R$ in supply node $i \in I$ we define $F_{ik}(\cdot)$ as a differentiable possibly nonlinear nondecreasing function of the investment x_{ik}^R, with $F_{ik}(0) = 0$, denoting the available capacity. In general $F_{ik}(\cdot)$ should reflect the fact that for most renewable resources like wind power, not all installed capacity is available for generation at all times. The availability is often subject to randomness, which we discuss in more detail in Section 3.5. The second stage problem for a non-renewable generator $k \in K^N$ is

$$\begin{aligned} \Pi_k^N(x_k^N) := \max_{y_k^N} \quad & \sum_{i \in I} (p_{ik}^N - c_{ik}^N) y_{ik}^N & & \\ \text{s.t.} \quad & y_{ik}^N \leq x_{ik}^N & (\beta_{ik}^N) \quad & \forall i \in I \\ & y_{ik}^N \geq 0 & & \forall i \in I. \end{aligned} \tag{3.4}$$

For a renewable generator $k \in K^R$ we have:

$$\begin{aligned} \Pi_k^R(x_k^R) := \max_{y_k^R} \quad & \sum_{i \in I} (p_{ik}^R - c_{ik}^R) y_{ik}^R & & \\ \text{s.t.} \quad & y_{ik}^R \leq F_{ik}(x_{ik}^R) & (\beta_{ik}^R) \quad & \forall i \in I \\ & y_{ik}^R \geq 0 & & \forall i \in I. \end{aligned} \tag{3.5}$$

The βs are the dual variables associated with the capacity constraints and represent the scarcity rents.

The produced electricity is transmitted from the supply nodes to the demand nodes by the Transmission System Operator (TSO). The TSO determines the optimal net flow f_j into node $j \in N \cup I$. Having $f_j < 0$ means there is a flow out of node j (which usually holds for the supply nodes). Power is transmitted through transmission lines $l \in L$. Each line in L runs from one node to another node. Each flow affects the capacities on all lines in the network either positively or negatively in accordance with Kirchhoff's Law; see for example Chao et al. (2000). Kirchhoff's Law is taken into account via the commonly used power transmission distribution factors (PTDF), which are given for a given network. For the network flows to be feasible, it must hold that the net load on line $l \in L$ must lie between the network capacities $-h_l$ and h_l. TSO's problem is to maximize the total value of the flows and can be written as:

$$
\begin{array}{lll}
\max\limits_{f} & \sum\limits_{j \in N \cup I} p_j^c f_j & \\
\text{s.t.} & \sum\limits_{j \in N \cup I} f_j = 0 & (\rho) \\
 & h_l - \sum\limits_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 & (\lambda_l^+) \quad \forall l \in L \\
 & h_l + \sum\limits_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 & (\lambda_l^-) \quad \forall l \in L.
\end{array}
\tag{3.6}
$$

The first constraint is the flow balance constraint with dual price ρ, and the last two constraints take care of the transmission capacity on the lines $l \in L$ and have dual prices λ_l^+ and λ_l^-, respectively.

Finally, two market clearing conditions should hold at equilibrium. First, in each supply node $i \in I$, where the demand is defined as $d_i = 0$, the flow out of this node can be at most equal to the total production in that node. In each demand node $n \in N$, where the supply is defined as $y_{nk}^N = 0 \; \forall k \in K^N$ and $y_{nk}^R = 0 \; \forall k \in K^R$, the flow into this node should be at least equal to the demand d_n (unless there is unsatisfied demand as we explain below). Perpendicular to those conditions, the market price p_j^c in each node $j \in N \cup I$ is determined. Second, there may be unsatisfied demand at node $j \in N \cup I$ which we denote by δ_j. Whenever the unsatisfied demand is positive, the market price in node j should be set to *VOLL*, the Value Of Lost Load, or to a high price-cap. In other words, *VOLL* can essentially serve as a price-cap and is in general taken as a high number, compared to regular electricity prices. The above relations are described by the following conditions:

$$
\begin{aligned}
0 \leq \sum_{k \in K^N} y_{jk}^N + \sum_{k \in K^R} y_{jk}^R + \delta_j + f_j - d_j \quad \perp \quad p_j^c \geq 0 \quad \forall j \in N \cup I \\
0 \leq VOLL - p_j^c \quad \perp \quad \delta_j \geq 0 \quad \forall j \in N \cup I.
\end{aligned}
\tag{3.7}
$$

The second stage problem now consists of the firms' problems (3.4) and (3.5), the TSO's problem (3.6), and the two types of market clearing conditions (3.7). For given x^N and x^R, a second stage equilibrium can be determined only when the relation between the prices p^N and p^R in relation to the consumer price p^c is given. This relation is fixed in a pricing scheme imposed by a regulator. Details are further discussed in the next section.

Given an equilibrium at the second stage, firms determine their first stage investment quantities based on the information from the second stage. In order to establish the connection between the two stages we write the KKT-optimality conditions that need to be solved for first stage optimality. The KKT-optimality conditions of the first stage problem given by (3.1) are:

$$0 \leq x_{ik}^{*N} \quad \perp \quad -\beta_{ik}^{*N} + \kappa_{ik}^{N} \geq 0 \qquad \forall i \in I, k \in K^N \tag{3.8}$$

These conditions follow from writing the Lagrangian function of (3.1) and taking derivatives with respect to x_{ik}^N, $i \in I, k \in K^N$. A detailed derivation of these first stage conditions can be found in Gürkan et al. (2013). x_{ik}^{*N} is the optimal investment quantity in non-renewable technology $k \in K^N$ in supply node $i \in I$ and β_{ik}^{*N} is the optimal scarcity rent taken from the second stage when $x = x^*$. For every $i \in I$, $k \in K^N$, at least one of the two sides in (3.8) should hold with equality, and hence there can only be a positive investment in technology $k \in K^N$ in supply node $i \in I$ if the corresponding optimal scarcity rent, β_{ik}^{*N}, covers the investment cost κ_{ik}^N. β_{ik}^{*N} can now be interpreted as the value of an additional unit investment in technology $k \in K^N$ in supply node $i \in I$.

The KKT-optimality conditions of the first stage problem given by (3.2) and (3.3) are:

$$\begin{array}{lll} 0 \leq x_{ik}^{*R} & \perp \quad -F_{ik}'(x_{ik}^{*R})\beta_{ik}^{*R} + \kappa_{ik}^{R} + \zeta_k^{*R} \geq 0 & \forall i \in I, k \in K^R \\ 0 \leq \zeta_k^{*R} & \perp \quad M_k^R - \sum_{i \in I} x_{ik}^{*R} \geq 0 & \forall k \in K^R. \end{array} \tag{3.9}$$

These conditions follow from writing the Lagrangian function of (3.2) subject to (3.3), and taking derivatives with respect to x_{ik}^R, $i \in I$, $k \in K^R$ and ζ_k^R, $k \in K^R$. x_{ik}^{*R} is the optimal investment quantity in renewable technology $k \in K^R$ in supply node $i \in I$ and β_{ik}^{*R} is the optimal scarcity rent taken from the second stage when $x = x^*$. The first condition in (3.9) means that at least one of the two sides should hold with equality and hence there can only be a positive investment in technology $k \in K^R$ in supply node $i \in I$ if $F_{ik}'(x_{ik}^{*R})\beta_{ik}^{*R}$ covers the sum of the investment cost κ_{ik}^R and the dual price, ζ_k^{*R}, associated with the ceiling. β_{ik}^{*R} is the optimal scarcity rent corresponding to a unit production, and $F_{ik}'(x_{ik}^{*R})$ is the change in production corresponding to a unit change in investment. Hence, $F_{ik}'(x_{ik}^{*R})\beta_{ik}^{*R}$ represents the scarcity rent of a unit investment when $x = x^*$. ζ_k^{*R} can be seen as the additional scarcity rent that comes with the ceiling, M_k^R. When the ceiling becomes binding, another, more expensive technology will be used to satisfy demand. As a consequence prices, and hence scarcity rents, increase with the additional rent ζ_k^{*R}. The second condition in (3.9) guarantees that ζ_k^{*R} is zero whenever the ceiling is not binding.

Finally, we analyze the equilibrium to the two stage game (3.1)-(3.7). The following theorem summarizes its key properties.

Theorem 3.1. *Consider the two-stage optimization problem defined by (3.1)-(3.7). At an equilibrium, the following holds.*
For investment in non-renewable technology $k \in K^N$ in supply node $i \in I$ to be positive, it must hold that

- $p_{ik}^{*N} - c_{ik}^N = \kappa_{ik}^N$
- $\beta_{ik}^{*N} = \kappa_{ik}^N$
- $y_{ik}^{*N} = x_{ik}^{*N}$.

For investment in renewable technology $k \in K^R$ in supply node $i \in I$ to be positive, it must hold that

- $F'_{ik}(x_{ik}^{*R})(p_{ik}^{*R} - c_{ik}^R) = \kappa_{ik}^R + \zeta_k^{*R}$
- $F'_{ik}(x_{ik}^{*R})\beta_{ik}^{*R} = \kappa_{ik}^R + \zeta_k^{*R}$
- $y_{ik}^{*R} = F_{ik}(x_{ik}^{*R})$.

Proof: Suppose the investment in non-renewable technology $k \in K^N$ in supply node $i \in I$ is positive, that is, $x_{ik}^{*N} > 0$. If $p_{ik}^{*N} - c_{ik}^N < \kappa_{ik}^N$, by $y_{ik}^{*N} \leq x_{ik}^{*N}$ firm k has a negative first stage profit in supply node i and is better off with no investments. On the other hand, if $p_{ik}^{*N} - c_{ik}^N > \kappa_{ik}^N$, firm k's investment goes to infinity and hence we have no equilibrium. Hence, $p_{ik}^{*N} - c_{ik}^N = \kappa_{ik}^N$. Then $\beta_{ik}^{*N} = \kappa_{ik}^N$ follows immediately from (3.8) and $x_{ik}^{*N} > 0$. $y_{ik}^{*N} = x_{ik}^{*N}$ must hold, since by complementary slackness in (3.5), $y_{ik}^{*N} < x_{ik}^{*N}$ implies $\beta_{ik}^{*N} = 0$ which contradicts our previous statement. The proof for renewable technology $k \in K^R$ is similar and is omitted. □

Note that $F'_{ik}(x_{ik}^{*R})(p_{ik}^{*R} - c_{ik}^R) = \kappa_{ik}^R + \zeta_k^{*R}$ can be interpreted as follows. On the left hand side, $p_{ik}^{*R} - c_{ik}^R$ is the net marginal revenue of production, and taking the derivative of production with respect to investment gives us the net marginal revenue of investment. At an equilibrium, this net marginal revenue of investment should be equal to the right hand side, which is the net marginal cost of investment including the additional scarcity rent for investment in case (3.3) is binding.

In the absence of environmental regulation to encourage investments in renewable technologies, both the price for non-renewable technologies and for renewable technologies are typically set at the consumer price. Together with the results in Theorem 3.1, this implies the following. In each supply node only a single technology, namely the one with the lowest sum of per unit production and investment cost, has a positive investment. The consumer price and hence the price for all technologies in that node will be set equal to this sum. In reality, whenever there is a non-renewable technology available in a node, none of the renewable technologies will have a positive investment because of its high investment cost. However, given that we may have different prices for both non-renewable and renewable technologies, we can incorporate some regulation that will set the price for renewable technologies higher as to create a financial incentive for investment in some renewable technology to be positive. We do this in the next section by introducing a renewable obligation that comes with a reward for producing with renewable technologies.

3.3 Introducing a Renewable Obligation and Tradable Green Certificates

We introduce a renewable obligation target in the electricity market investment model that was introduced in the previous section. An obligation target, denoted by ϕ, is set by the regulator. $\phi \in [0, 1]$ is the minimum proportion of total electricity production that should come from renewable resources. Let $Y^N = \sum_{i \in I} \sum_{k \in K^N} y_{ik}^N$ and $Y^R = \sum_{i \in I} \sum_{k \in K^R} y_{ik}^R$ be the total production using non-renewable and renewable plants, respectively. Given the obligation target it should hold that

$$\frac{Y^R}{Y^R + Y^N} \geq \phi. \tag{3.10}$$

Production with renewable resources is in general more expensive (when considering both investment and production cost) than production with non-renewable resources. The obligation (3.10) forces producers to replace non-renewable production with renewable production and thus comes with a certain additional cost. We let ν be the cost incurred with a unit increase of renewable production. More specifically, ν can be seen as the dual variable to (3.10) and represents the mark-up renewable generators should get in order to increase their production by one unit. These mark-ups are given to the firms through certificates. For each unit production, a renewable certificate is obtained and ν should thus be the value of such a certificate. Certificates can either be traded on a secondary market, or sold to the regulator to show compliance to the obligation. When trading is considered, ν would be the fair price for each certificate. When, like we assume, the regulator rewards firms for owning certificates, ν is the reward per certificate firms get paid at the end of each period, typically a year. It is thus referred to as the certificate price (from the regulator's perspective) or reward (from the firm's perspective). Rewriting (3.10) with ν as its dual variable, leads to the following complementarity condition that should hold at equilibrium:

$$0 \leq \sum_{k \in K^R} (1-\phi) Y_k^R - \sum_{k \in K^N} \phi Y_k^N \qquad \perp \qquad \nu \geq 0, \tag{3.11}$$

where $Y_k^R := \sum_{i \in I} y_{ik}^R$ for $k \in K^R$ and $Y_k^N := \sum_{i \in I} y_{ik}^N$ for $k \in K^N$. These aggregate variables per technology will be used in the remainder of the chapter to simplify notation.

We next consider the effect of rewards on the nodal prices. The electricity price in each supply node is usually set by the technology that produces with the highest marginal production cost. Since the fuel cost and hence the unit production cost (as opposed to the unit investment cost) of renewable technologies will in general be very low, without loss of generality we can assume that the electricity price will be set by a non-renewable technology. We refer to this price as the base price in node $i \in I$ and denote it by p_i^N. When power is bought from a non-renewable generator $k \in K^N$, the price paid per unit thus equals $p_{ik}^N = p_i^N$. When a renewable generator $k \in K^R$ sells power, it sells both the power (at price p_i^N) to the consumer and the certificate (at price ν) to the regulator, and should therefore be paid $p_{ik}^R = p_i^N + \nu$.

In electricity markets consumers typically pay a fixed consumer price that is independent of the resource. If consumers would only pay the electricity price p_i^N in every node i, the rewards paid to the firms could not be covered. The regulator therefore regulates the consumer price and adds a mark-up on top of p_i^N. The mark-up is set in such a way that the additional income covers the rewards paid to firms for owning certificates (that is, to guarantee revenue adequacy). Since a part ϕ of the total production should come from renewable resources, given that the total production equals $Y (= Y^N + Y^R)$, the total renewable production should be ϕY. This is also the number of times a reward should be paid to the firms. Hence, the mark-ups for the consumer price should cover $\phi Y \nu$. Furthermore, we want the mark-up to be equal in each node i; that is, although the electricity price can be different in each node due to the network structure, the certificate price and thus the mark-up are the result of putting a constraint on the entire market and hence should not be different for different nodes. Assuming that all the produced power will be sold, it should hold that the mark-up Δp^c is the solution of $\Delta p^c Y = \phi Y \nu$, resulting in a consumer price equal to $p_i^c = p_i^N + \phi \nu$, $i \in I$. This is imposed in the following pricing scheme:

$$\begin{aligned} p_{ik}^N &= p_i^N && \forall i \in I, k \in K^N \\ p_{ik}^R &= p_i^N + \nu && \forall i \in I, k \in K^R \\ p_i^c &= p_i^N + \phi \nu && \forall i \in I. \end{aligned} \tag{3.12}$$

Given this pricing scheme, we can express both p_{ik}^N and p_{ik}^R in (3.4) and (3.5) in terms of p_i^c; that is, $p_{ik}^N = p_i^c - \phi \nu$ for $k \in K^N$ and $p_{ik}^R = p_i^c + (1-\phi)\nu$ for $k \in K^R$. Now a solution of the entire second stage problem can be obtained by solving the KKT conditions of (3.4), (3.5), (3.6), the market clearing conditions (3.7), and the obligation (3.11) for given x:

$$\begin{aligned} 0 \le \beta_{ik}^{*N} - p_i^{*c} + \phi \nu^* + c_{ik}^N &\perp y_{ik}^{*N} \ge 0 && \forall i \in I, k \in K^N \\ 0 \le \beta_{ik}^{*R} - p_i^{*c} - (1-\phi)\nu^* + c_{ik}^R &\perp y_{ik}^{*R} \ge 0 && \forall i \in I, k \in K^R \\ 0 \le x_{ik}^N - y_{ik}^{*N} &\perp \beta_{ik}^{*N} \ge 0 && \forall i \in I, k \in K^N \\ 0 \le F_{ik}(x_{ik}^R) - y_{ik}^{*R} &\perp \beta_{ik}^{*R} \ge 0 && \forall i \in I, k \in K^R \\ 0 \le VOLL - p_j^{*c} &\perp \delta_j^* \ge 0 && \forall j \in N \cup I \\ 0 \le \sum_{k \in K^N} y_{jk}^{*N} + \sum_{k \in K^R} y_{jk}^{*R} + \delta_j^* + f_j^* - d_j &\perp p_j^{*c} \ge 0 && \forall j \in N \cup I \\ 0 \le \sum_{i \in I} \sum_{k \in K^R} (1-\phi) y_{ik}^{*R} - \sum_{i \in I} \sum_{k \in K^N} \phi y_{ik}^{*N} &\perp \nu^* \ge 0 && \\ 0 \le h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j^* &\perp \lambda_l^{*+} \ge 0 && \forall l \in L \\ 0 \le h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j^* &\perp \lambda_l^{*-} \ge 0 && \forall l \in L \\ \sum_{l \in L} PTDF_{l,j}(\lambda_l^{*+} - \lambda_l^{*-}) + p_j^{*c} - \rho^* = 0 & && \forall j \in N \cup I \\ \sum_{j \in N \cup I} f_j^* = 0. & && \end{aligned} \tag{3.13}$$

Note that (3.13) is in fact equivalent to a single optimization problem, a so-called optimal power flow (OPF) problem. The OPF problem shows us explicitly how certificates affect the cost structure. The OPF problem is:

$$
\begin{aligned}
&Z(x) := \\
&\quad \min_{y,\delta,f} \sum_{i\in I}\sum_{k\in K^N} c_{ik}^N y_{ik}^N + \sum_{i\in I}\sum_{k\in K^R} c_{ik}^R y_{ik}^R + VOLL \sum_{j\in N\cup I} \delta_j \\
&\quad \text{s.t. } \begin{array}{lll}
y_{ik}^N \le x_{ik}^N & (\beta_{ik}^N) & \forall i \in I, k \in K^N \\
y_{ik}^R \le F_{ik}(x_{ik}^R) & (\beta_{ik}^R) & \forall i \in I, k \in K^R \\
\sum_{k\in K^N} y_{jk}^N + \sum_{k\in K^R} y_{jk}^R + \delta_j + f_j \ge d_j & (p_j^c) & \forall j \in N\cup I \\
\sum_{i\in I}\sum_{k\in K^R} (1-\phi) y_{ik}^R - \sum_{i\in I}\sum_{k\in K^N} \phi y_{ik}^N \ge 0 & (\nu) & \\
\sum_{j\in N\cup I} f_j = 0 & (\rho) & \\
\sum_{j\in N\cup I} PTDF_{l,j} f_j \le h_l & (\lambda_l^+) & \forall l \in L \\
\sum_{j\in N\cup I} PTDF_{l,j} f_j \ge -h_l & (\lambda_l^-) & \forall l \in L \\
y_{ik}^N \ge 0 & & \forall i \in I, k \in K^N \\
y_{ik}^R \ge 0 & & \forall i \in I, k \in K^R \\
\delta_j \ge 0 & & \forall j \in N\cup I.
\end{array}
\end{aligned}
\tag{3.14}
$$

A solution (y^*, δ^*, f^*) of the OPF problem satisfies KKT-conditions (3.13) for some $\beta^*, p^{*c}, \nu^*, \rho^*, \lambda^{*+}$, λ^{*-}. More details on the derivation for the base case model can be found in Boucher and Smeers (2001) and Gürkan et al. (2013); ours is a straightforward extension. Solving the OPF problem gives a second stage equilibrium for given x. The entire investment problem can now be solved by finding an equilibrium to the second stage problem (3.14) such that $x = x^*$, together with the first stage optimality conditions consisting of (3.8) and (3.9).

In Theorem 3.1 we gave properties of x^*, y^*, p^*, and β^* at the equilibrium. Given the pricing scheme (3.12), we can now replace the prices and analyze the equilibrium in case of a renewable obligation. Recall that for a non-renewable technology $k \in K^N$ to have positive investments in supply node $i \in I$ it should hold that $p_{ik}^{*N} - c_{ik}^N = \kappa_{ik}^N$. Rewriting using (3.12) gives $p_i^{*c} = c_{ik}^N + \kappa_{ik}^N + \phi\nu^*$. Furthermore we have $\beta_{ik}^{*N} = \kappa_{ik}^N$ and $y_{ik}^{*N} = x_{ik}^{*N}, k \in K^N$, $i \in I$. Only one non-renewable technology will have a positive investment and will thus set the price for non-renewable technologies. For a renewable technology $k \in K^R$ to have positive investments in supply node $i \in I$, it must hold that $F'_{ik}(x_{ik}^{*R})(p_{ik}^{*R} - c_{ik}^R) = \kappa_{ik}^R + \zeta_k^{*R}$; rewriting using our pricing scheme

(3.12), we get

$$p_i^{*c} = c_{ik}^R + \frac{1}{F'_{ik}(x_{ik}^{*R})}(\kappa_{ik}^R + \zeta_k^{*R}) - (1-\phi)\nu^*. \tag{3.15}$$

In addition, when investment is positive, we have $F'_{ik}(x_{ik}^{*R})\beta_{ik}^{*R} = \kappa_{ik}^R + \zeta_k^{*R}$, and $y_{ik}^{*R} = F_{ik}(x_{ik}^{*R})$. Due to the obligation, ν^* will now be set such that at least a fraction ϕ will be produced with renewable technologies, and hence at least one renewable technology must have a positive investment. Depending on the caps on investment quantities M_k^R, $k \in K^R$, multiple renewable technologies can be in the equilibrium mixture. All capacity of the cheapest renewable technology (in terms of the sum of their production and investment cost), let's say $k^0 \in K^R$, would be used up first. If that is not sufficient to satisfy the obligation, k^0 would invest up to his maximum capacity and $\zeta_{k^0}^{*R}$ would become positive. The next cheapest renewable technology will be used and as such the consumer price in node i, p_i^{*c}, will increase. $\zeta_{k^0}^{*R}$ will be determined such that the equality in (3.15) for technology k^0 still holds and can thus be seen as the additional rent for investment in technology k^0 due to the ceiling.

Since only the cheapest firms invest at the equilibrium, the deterministic equilibrium can simply be based on all the parameters and is thus quite straightforward. In reality, broader mixtures of technologies are used, which is the result of daily uncertainties like demand and weather fluctuations. We therefore continue our analysis of the effects of the renewable obligation in Sections 3.5 and 3.6 where we deal with a stochastic version of the model and carry out a numerical study. Before doing so, we first extend the renewable obligation to the case in which different renewable technologies are eligible for a different number of certificates per unit production.

3.4 Introducing a Banding System

The previously introduced model was applicable to the UK system until the 1st of April 2009, when the Renewable Obligation Order 2009 became effective.[4] In this regulatory document a banding system was added to the renewable obligation. In the old system, a unit production with a renewable technology was eligible for one renewable energy certificate and hence would receive the certificate price for each unit production. In the new system, renewable certificates are banded according to technology. This means that different re-

[4]See http://www.legislation.gov.uk/uksi/2009/785/contents/made?view=plain.

newable technologies are eligible for a different number of certificates. The main reason for introducing bandings is to encourage investments in less established technologies (by giving them more certificates per unit production). The original renewable obligation failed to give the right financial incentives as it tended to mainly benefit onshore wind power and landfill gas, as noted by Meyer (2003). As argued by Toke (2011) and Wood and Dow (2011), investments in these more established technologies are limited not by the lack of financial incentives, but mostly because of landscape protection, public opposition, and space. Therefore, investment in less developed technologies will be necessary in order to be able to achieve the ambitious renewable energy targets.

In the new system onshore wind is used as the reference for bandings. A unit production with onshore wind is rewarded with one certificate, less established technologies like offshore wind and geothermal are rewarded with 1.5 and 2 certificates, respectively, and more established technologies like sewage gas and landfill gas are rewarded with only 0.5 and 0.25 certificates, respectively. The regulator may change these coefficients from time to time when a different support is desirable. This has recently been done for offshore wind, which now gets 2 certificates per unit production until 2014.

In this section we first incorporate the UK banding system into the existing model. We introduce a pricing scheme and analyze the advantages and drawbacks of the new system. In Section 3.4.2 we propose an alternative that is closer to the Italian banding system in order to overcome some of these drawbacks.

3.4.1 The UK Banding System

With the banding system as introduced in the UK, the obligation shifts from one on the renewable production to one that is expressed as the number of certificates that should be presented by the entire market at the end of each period. Hence, with the new system, obligation constraint (3.11) should be replaced by a condition of the following form:

$$0 \leq \sum_{k \in K^R} \alpha_k Y_k^R - R \qquad \perp \qquad \nu \geq 0, \tag{3.16}$$

where R is the number of certificates all firms together have to present, and α_k is the number of certificates firm $k \in K^R$ receives for producing one unit of electricity with renewable technology k.

As explained in great detail in the Renewable Obligation Order 2009, in order to determine R, the UK regulator performs two calculations estimating the number of certificates that should or can potentially be issued. Whichever calculation gives the highest estimate will be set as the obligation target R. The first calculation, Calculation A, is based on a fixed target representing the number of certificates firms should produce per unit of electricity produced. This target is in fact the original obligation target ϕ that was used in system (3.11). The total number of certificates that should be issued based on this fixed target is simply ϕ times the expected total electricity production and will be defined as A.

In the second calculation, Calculation B, the number of certificates that is likely to be issued, given the existing renewable production capacity and the expected new built capacity, is estimated. In addition a headroom of 8% is added. The outcome is defined as B.

In practice this means that if $A > B$, it is expected that the existing and expected new built capacity will not be sufficient to reach the original obligation target ϕ that was used for Calculation A. Hence, the target is set at A to oblige firms to install additional renewable capacity. If $B > A$, it means that there is already sufficient existing plus expected new built capacity in the system. When this happens, the original target ϕ may be met quite easily and as a consequence the value of certificates may drop to zero, resulting in an unfavorable situation for the renewable generators. Setting the obligation target at B should avoid this. In order to provide even more security and to give extra incentives, an additional headroom of 8% is added to the originally computed expected number of certificates that is likely to be issued.

After the target $R = max\{A, B\}$ is determined, the regulator obliges all firms to produce a certain number of certificates per unit production. If $A > B$, each firm will need to submit ϕ certificates per unit production. If $B > A$ each firm will have to submit $\phi^B = \frac{\phi A}{B}$ certificates per unit production. Since we assume perfect competition and deal with certificates that can be traded on a secondary market, this obligation per firm can also be seen as one for the entire market. We let ϕ^{UK} be the target on certificates based on the calculations above (either ϕ or ϕ^B), which means that R, the total number of certificates that should be presented, equals ϕ^{UK} times the total production. Hence, replacing R in (3.16), the obligation can be written as

$$0 \leq \sum_{k \in K^R} (\alpha_k - \phi^{UK}) Y_k^R - \sum_{k \in K^N} \phi^{UK} Y_k^N \qquad \perp \qquad \nu \geq 0. \tag{3.17}$$

Since different technologies are rewarded with a different number of certificates, prices paid to the generators will depend on the technology used. Therefore, we have to adjust the pricing scheme (3.12) introduced under the original renewable obligation. Per unit production in any supply node $i \in I$, a renewable technology $k \in K^R$ is given α_k certificates. Since, when selling the electricity, firms also sell their certificates, renewable technology $k \in K^R$ is then paid the base price, p_i^N, plus α_k times the certificate price ν. In the corresponding pricing scheme we get that $p_{ik}^N = p_i^N$, $k \in K^N$, $i \in I$ and $p_{ik}^R = p_i^N + \alpha_k \nu$, $k \in K^R$, $i \in I$. With a consumer price similar to the one in (3.12), namely $p_i^c = p_i^N + \phi^{UK}\nu$, $i \in I$, it turns out that mark ups paid by consumers equal the rewards paid to firms for owning certificates. That is, renewable technologies are paid $\nu \sum\limits_{k \in K^R} \alpha_k Y_k^R$, which by (3.17) equals $\nu\phi^{UK}Y$. It can easily be seen that $\nu\phi^{UK}Y$ is the total mark ups paid by consumers, and hence rewards paid to firms are covered. This means the pricing scheme is revenue adequate. Summarizing, the adjusted pricing scheme for the UK banding system becomes

$$\begin{aligned} p_{ik}^N &= p_i^N && \forall i \in I, k \in K^N \\ p_{ik}^R &= p_i^N + \alpha_k \nu && \forall i \in I, k \in K^R \\ p_i^c &= p_i^N + \phi^{UK}\nu && \forall i \in I. \end{aligned} \tag{3.18}$$

Note that ideally it would hold that, with the obligation on certificates, a fraction ϕ^{UK} of the total production comes from renewable resources like in the old obligation system. However, since there is no one-to-one relationship between renewable production and certificates anymore, this is not necessarily the case. In case a major part of the obligation is satisfied by a technology with a high banding coefficient, the actual renewable production may be (much) lower than the desired target on production and as a consequence the banding system could result in a more polluting mixture of technologies. In general, Calculation B may set the target on certificates a bit higher than the original obligation ϕ, but when there are technologies with a high banding coefficient, one may not guarantee that the original target on renewable production is met unless the target on certificates is increased accordingly.

We will come back on this issue in our numerical study. In the next section, we propose an alternative system that combines features of the original obligation and the UK banding system.

3.4.2 An Alternative Banding System

As the UK banding system does not necessarily guarantee that the obligation target on renewable production (3.11) is satisfied and may even result in a more polluting mixture, we propose an alternative banding system. Unlike in the UK banding system where the obligation shifts to one on certificates, the obligation will be imposed on production in the same way it was done in the original renewable obligation (that is, as in (3.11) as opposed to (3.17)). On the other hand, certificates will still be handed out based on the pre-specified banding coefficients. Since in this case we no longer have a one-to-one correspondence between a unit of renewable production and a certificate, and since the target is no longer on certificates like in the UK banding system, the dual price ν in (3.11) no longer represents the value of a certificate; it just represents the value of a unit production with a renewable resource. This will have a consequence for the reward per certificate and hence for the pricing scheme.

As we have seen, under the previously discussed obligation policies consumers pay a mark-up that covers the rewards the regulator pays to the firms for owning certificates; in other words, the proposed pricing schemes are revenue adequate for the regulator. We would like this revenue adequacy to hold in the alternative system as well. In the UK banding system, in the pricing scheme (3.18), the consumer price was set at $p_i^c = p_i^N + \phi\nu$ in each supply node $i \in I$. We adopt this price in our alternative system as this price does not depend on the α_ks and hence on the technologies used, which is typically the case in electricity markets. With this price the total income from mark-ups paid by consumers equals

$$\phi\nu \left(\sum_{k \in K^N} Y_k^N + \sum_{k \in K^R} Y_k^R \right) = \nu \sum_{k \in K^R} Y_k^R.$$

The latter equality holds by (3.11); that is, either $\nu = 0$ or the left hand side inequality in (3.11) holds with equality. The total income has to be divided over the total amount of certificates on the market, which is $\sum_{k \in K^R} \alpha_k Y_k^R$. Therefore, instead of ν, we are going to pay firms an adjusted certificate price $\tilde{\nu}$, which is determined as follows:

$$\tilde{\nu} = \frac{\nu \sum_{k \in K^R} Y_k^R}{\sum_{k \in K^R} \alpha_k Y_k^R}.$$

As mentioned above, due to the different interpretation of ν in (3.11), such an adjustment was not necessary in the UK banding system. In the investment model we incorporate the above condition as the following equality that will have to hold at the second stage:

$$\sum_{k \in K^R} (\tilde{\nu}\alpha_k - \nu) Y_k^R = 0. \tag{3.19}$$

This condition, that determines the value of $\tilde{\nu}$, poses a non-linearity. While, unlike in the other models, we cannot incorporate this into an OPF problem formulation, we can still deal with the entire problem as a mixed complementarity problem by solving the entire set of KKT-optimality conditions. Another consequence of the non-linearity is a longer computational time in numerical experiments that will be carried out in Section 3.6. The resulting pricing scheme in the alternative system becomes

$$\begin{aligned} p_{ik}^N &= p_i^N && \forall i \in I, k \in K^N \\ p_{ik}^R &= p_i^N + \alpha_k \tilde{\nu} && \forall i \in I, k \in K^R \\ p_i^c &= p_i^N + \phi\nu && \forall i \in I. \end{aligned} \tag{3.20}$$

It can easily be seen with (3.11) and (3.19) that this new scheme is revenue adequate for the alternative banding system. In practice, it may not be straightforward how to implement this scheme as the value of $\tilde{\nu}$, and hence the unit reward paid to renewable producers, depends on the (realized) renewable production. A regulator could either use contracts in which the final unit reward per certificate is determined ex-post, at the end of each contract period, or determine adjusted rewards ex-ante, based on historical production data.

Although this alternative system guarantees that the original obligation target on production is satisfied, there are a few drawbacks to this system. First of all, the price of a certificate is no longer determined by the secondary trading market. The regulator has to intervene and buy certificates from firms at an adjusted price that is influenced by both the technology mixture and the mark-ups paid by consumers. The intervention in the trading process is likely to come with a certain administrative burden. Second, as we will see in more detail in our numerical study, the consumer price is much more sensitive to the technology mixture and in particular is higher when a large share of the production is done with a technology with a banding coefficient higher than 1. Furthermore, the consumer price could even increase as a result of a cost reduction (due to for example innovation) of a technology; this, we do not observe in the UK banding system. However, as we will show in our numerical study, in the UK banding system cost reductions can potentially lead to more polluting technology mixtures.

3.5 Introducing Uncertainty

The previously introduced models all assume a deterministic world. In reality however, demand can be uncertain due to seasonality and daily changes in, for example, weather patterns, and also the output of renewable power plants may vary from day to day, hour to hour. For example wind turbines are depending on daily weather conditions and influenced by the actual wind speed. A unit investment in wind energy does not mean we can produce a given amount of power at all times. We refer to this uncertainty as the uncertainty in availability of capacity.

The way we deal with uncertainty can be explained as follows. At the first stage, only the underlying probability distributions (which can be simply based on past empirical data) of the demand and availability of capacity are known; firms have to make decisions on their optimal investment quantities without knowing the outcome of the random variables. At the second stage the realizations are revealed to the firms. When firms make their first stage decision, they view the second stage as a short-run process that repeats itself over and over again. Each realization of the random variable can for example be seen as the realization associated with a particular day (or even an hour). On every day of the year there is an outcome, and the first stage decision is taken at the beginning of the year. Hence, the first stage decision is a long-term decision. When making the decision, we take either the expectation or the sample average (in case we use a sampling technique) over all days and determine the investment level based on the expected or sample averaged second stage outcome.

When moving to the stochastic setting, a couple of modifications to the proposed models are needed. First, consider the basic two-stage model consisting of the first stage problem consisting of (3.1), (3.2), and (3.3) and the second stage problems (3.4), (3.5), (3.6), and (3.7). At the first stage, we assume the available renewable capacity at the second stage and the second stage demand to be unknown to the firms. At the second stage, we introduce $\omega \in \Omega$, a random vector in the space of possible outcomes Ω, containing the outcomes of both random available capacities and random demand. These outcomes are assumed to have some joint distribution Ψ. Outcomes are assumed to be revealed to firms at the second stage. To incorporate uncertain availability of capacity, for $k \in K^R$, $i \in I$, we define $F_{ik}(x, \omega)$ as some differentiable random function of the investment amount x of technology k in supply node i at realization $\omega \in \Omega$. For given $\omega \in \Omega$, $F_{ik}(x, \omega)$ is the realized available capacity in technology $k \in K^R$ in node $i \in I$. Uncertain demand in demand node $n \in N$ at realization $\omega \in \Omega$ is denoted by $d_n(\omega)$. At the first stage, given the probability distributions (or historical data) of the random variables at the second stage, long-term profits are maximized and the corresponding optimal investment quantities are determined. Long-term profits now consist of the expected second stage profits minus the first stage investment cost. For non-renewable generator $k \in K^N$ the first stage problem becomes:

$$\max_{x_k^N \geq 0} \quad E_\omega \left[\sum_{i \in I} (p_{ik}^N(\omega) - c_{ik}^N) y_{ik}^N(x_k^N, \omega) \right] - \sum_{i \in I} \kappa_{ik}^N x_{ik}^N. \tag{3.21}$$

For renewable generator $k \in K^R$ the first stage problem becomes:

$$\begin{aligned} \max_{x_k^R \geq 0} \quad & E_\omega \left[\sum_{i \in I} (p_{ik}^R(\omega) - c_{ik}^R) y_{ik}^R(x_k^R, \omega) \right] - \sum_{i \in I} \kappa_{ik}^R x_{ik}^R \\ \text{s.t.} \quad & \sum_{i \in I} x_{ik}^R \leq M_k^R \qquad\qquad\qquad\qquad\qquad\qquad (\zeta_k^R). \end{aligned} \tag{3.22}$$

The second stage for given x and any $\omega \in \Omega$ is given by the OPF problem (3.14) with ω attached to all variables, d, and F, and without the fourth constraint that deals with the renewable obligation. The renewable obligation constraint is instead going to be imposed as a first stage constraint, as we explain below. For given $x = (x^N, x^R)$ and in any $\omega \in \Omega$, we thus solve the following second stage problem:

$$
\begin{aligned}
& Z(x,\omega) := \\
& \min_{y(\omega),\delta(\omega),f(\omega)} \sum_{i\in I}\sum_{k\in K^N} c^N_{ik} y^N_{ik}(\omega) + \sum_{i\in I}\sum_{k\in K^R} c^R_{ik} y^R_{ik}(\omega) + VOLL \sum_{j\in N\cup I} \delta_j(\omega) \\
& \quad \text{s.t.} \quad
\begin{array}{lll}
y^N_{ik}(\omega) \le x^N_{ik} & (\beta^N_{ik}(\omega)) & \forall i \in I, k \in K^N \\
y^R_{ik}(\omega) \le F_{ik}(x^R_{ik},\omega) & (\beta^R_{ik}(\omega)) & \forall i \in I, k \in K^R \\
\sum_{k\in K^N} y^N_{jk}(\omega) + \sum_{k\in K^R} y^R_{jk}(\omega) + & & \\
\delta_j(\omega) + f_j(\omega) \ge d_j(\omega) & (p^c_j(\omega)) & \forall j \in N\cup I \\
\sum_{j\in N\cup I} f_j(\omega) = 0 & (\rho(\omega)) & \\
\sum_{j\in N\cup I} PTDF_{l,j} f_j(\omega) \le h_l & (\lambda^+_l(\omega)) & \forall l \in L \\
\sum_{j\in N\cup I} PTDF_{l,j} f_j(\omega) \ge -h_l & (\lambda^-_l(\omega)) & \forall l \in L \\
y^N_{ik}(\omega) \ge 0 & & \forall i \in I, k \in K^N \\
y^R_{ik}(\omega) \ge 0 & & \forall i \in I, k \in K^R \\
\delta_j(\omega) \ge 0 & & \forall j \in N\cup I.
\end{array}
\end{aligned}
\tag{3.23}
$$

The stochastic version of the basic model now consists of (3.21) for all $k \in K^N$ and (3.22) for all $k \in K^R$ at the first stage and (3.23) for all $\omega \in \Omega$ at the second stage.

When introducing a renewable obligation and a banding system, we need to make a few adjustments to the basic model. In all three systems, we replace the second stage obligation by its stochastic equivalent that is going to be imposed ex-ante, on the expected (or in numerical experiments sample-averaged) yearly production. This means that from day to day the obligation may be violated, but (in expectation) the end of the year target should be met. As such, we are interested in the ex-ante certificate price which depends on the underlying distribution (or historical data) of the random variable, but is fixed over the year. Notably, we replace (3.11), (3.17), and (3.19) with (3.24), (3.25), and (3.26), respectively; see below.

In principle, it is possible to consider a daily obligation as an alternative. The obligation should then be a second stage constraint (omitting the expectation) and each day (that is, in each realization) there could be a different certificate price. This implies that the certificate price will be fluctuating over the days. In this chapter, we focus on having the obligation constraint over a certain period, since this is what is applied in reality (see for example Bertoldi and Huld (2006) and van der Linden et al. (2005)).

In addition to replacing the obligation, we also need to adjust the pricing schemes (3.12), (3.18), and (3.20) in the original obligation system, the UK banding system, and the alternative banding system, respectively. We simply add ωs to the prices. With these modified pricing schemes, in expectation the mark-ups paid by consumers cover the rewards. However, due to the uncertain outcome at the second stage, ex-post there may exist gaps between daily rewards to be paid and daily income from mark-ups. Focusing on such situations in more detail may be interesting for regulators aiming at daily revenue adequacy; however, it is outside the scope of this thesis.

Finally, as a consequence of moving the obligation from the second to the first stage problem, in each of the systems the objective function of the OPF problem (3.23) needs a modification. All the necessary modifications to the obligation and the objective function of (3.23) are summarized below.

- **No banding**:
  - Replace (3.11) at the second stage by the first stage condition

$$0 \leq E_\omega \left[\sum_{k \in K^R} (1-\phi) Y_k^R(\omega) - \sum_{k \in K^N} \phi Y_k^N(\omega) \right] \quad \perp \quad \nu \geq 0. \tag{3.24}$$

  - Replace the objective function of (3.23) by

$$\begin{aligned} \min_{y(\omega),\delta(\omega),f(\omega)} & \sum_{i \in I} \sum_{k \in K^N} (c_{ik}^N + \phi\nu) y_{ik}^N(\omega) + \\ & \sum_{i \in I} \sum_{k \in K^R} (c_{ik}^R - (1-\phi)\nu) y_{ik}^R(\omega) + VOLL \sum_{j \in N \cup I} \delta_j(\omega). \end{aligned}$$

- **UK banding**:
  - Replace (3.17) at the second stage by the first stage condition

$$0 \leq E_\omega \left[\sum_{k \in K^R} (\alpha_k - \phi^{UK}) Y_k^R(\omega) - \sum_{k \in K^N} \phi^{UK} Y_k^N(\omega) \right] \quad \perp \quad \nu \geq 0. \tag{3.25}$$

  - Replace the objective function of (3.23) by

$$\min_{y(\omega),\delta(\omega),f(\omega)} \sum_{i \in I} \sum_{k \in K^N} (c_{ik}^N + \phi^{UK}\nu) y_{ik}^N(\omega) +$$

$$\sum_{i\in I}\sum_{k\in K^R}(c_{ik}^R-(\alpha_k-\phi^{UK})\nu)y_{ik}^R(\omega)+VOLL\sum_{j\in N\cup I}\delta_j(\omega).$$

- **Alternative banding**:
  - Replace (3.11) at the second stage by the first stage condition (3.24).
  - Replace (3.19) at the second stage by the first stage condition

$$E_\omega\left[\sum_{k\in K^R}(\tilde{\nu}\alpha_k-\nu)Y_k^R(\omega)\right]=0. \tag{3.26}$$

  - Replace the objective function of (3.23) by

$$\min_{y(\omega),\delta(\omega),f(\omega)}\sum_{i\in I}\sum_{k\in K^N}(c_{ik}^N+\phi\nu)y_{ik}^N(\omega)+$$
$$\sum_{i\in I}\sum_{k\in K^R}(c_{ik}^R-\alpha_k\tilde{\nu}+\phi\nu)y_{ik}^R(\omega)+VOLL\sum_{j\in N\cup I}\delta_j(\omega).$$

3.6 Numerical Study

In this section we analyze the effects of the different renewable obligation systems on investments, prices, and CO_2 emissions in a numerical framework. In order to keep things tractable and solvable in a reasonable amount of time, we consider a small system. The setting is rigorously simplified in terms of network effects, but nonetheless we can draw some conclusions about possible side effects of the systems and make a comparison between them.

We are going to focus on three different renewable obligation policies:

- **No banding**: There is no banding mechanism, meaning all α_ks for renewable technologies are equal to 1.
- **UK banding**: The banding mechanism that is currently applied in the UK is imposed, under the assumption that Calculation A sets the target; the obligation is expressed as the number of certificates per MWh of power produced.
- **Alternative banding**: The banding mechanism as proposed in Section 3.4.2 is imposed; the obligation is expressed as the amount of power that should come from a renewable resource per MWh of power produced, and rewards are adjusted in order to guarantee long term revenue adequacy.

First, for different target levels we depict investments, prices, and CO_2 emissions in order to analyze the effects of the different systems and to get insight in their advantages and disadvantages. Second, it is reasonable to expect that after a technology is given support, more innovation and development will take place and subsequently that investment costs decrease. We therefore analyze the effects of a cost reduction of one of the technologies in the second part of this section.

We utilize the MCPs consisting of the first and second stage KKT optimality conditions as introduced in the previous sections. In order to deal with uncertainty, we use a sampling technique. We generate 3000 random samples from given probability distributions for the available renewable capacities and demand. We then simultaneously solve all second stage KKT conditions in 3000 realizations and the first stage KKT conditions in which we replace expectations by sample averages over all realizations. The obtained large sized MCPs are programmed in GAMS and solved using the PATH solver, see Ferris and Munson (2000). Using a 300MHz Pentium-II with 1 GB RAM, computation times are approximately two hours for each instance we solve.

3.6.1 Experimental Data

We consider a single node and hence assume there is no limited network capacity on the transmission lines. For a thorough analysis of investment under uncertainty in transmission capacity in the UK electricity market, we refer to van der Weijde and Hobbs (2012). We instead focus on the direct effects of obligation policies in absence of network limitations. We consider five firms, each having a unique technology at their disposal. Non-renewable technologies coal (Coal) and closed cycle gas turbine (CCGT) are used by firms 1 and 2, respectively. Renewable technologies onshore wind (ONW), offshore wind (OFFW), and landfill gas (LFG), are used by firms 3, 4, and 5, respectively. In addition there are two demand nodes, nodes 6 and 7.

Table 3.1 contains the characteristics of the technologies, consisting of per unit production costs (c_k), investment costs (κ_k), tons of CO_2 emission per unit production (e_k), and the banding coefficient (α_k).

The CO_2 emission per unit production, e_k, is given to indicate how polluting the technologies are. The cost figures are in Pounds and based on data in MottMacDonald (2010). It is quite common in numerical studies to work with levelized cost of investment; that is, the investment cost that is needed to produce 1 MWh of electricity, taking into account that not every MW installed

Table 3.1: Characteristics of the technologies.

	Coal	CCGT	ONW	OFFW	LFG
c_k	22.1	50.2	0	0	21.1
κ_k	30.24	12.96	25.62	53.48	24.78
e_k	1	0.35	0	0	0.2
α_k	0	0	1	2	0.25

will be available at all times. We ignore this when defining the investment costs that play a role at the first stage, and will thus work with the real investment cost that is involved with having 1 MW of capacity installed. The fact that not all installed capacity is available for the full 8760 hours in the year is taken into account at the second stage via the function $F(\cdot)$ that we specify below.

Demand $d_n(\omega)$ in demand nodes $n = 6,7$ are independent and are sampled from uniform distributions with lower bound a_n and upper bound b_n as in Table 3.2.

Table 3.2: Parameters for uniform demand distribution.

	a_n	b_n
6	8	12
7	15	20

The available wind and landfill gas capacities are also randomly distributed. As a function representing the realized available capacity of technology k we take $F_k(x,\omega) = \theta_k(\omega)x$, where for each renewable technology $\theta_k(\omega)$ is sampled from a uniform distribution with lower bound a_k and upper bound b_k, $k = 3,4,5$, as in Table 3.3. We assume onshore and offshore wind are fully correlated, but assume independence between wind and landfill gas realizations. We chose uniform distributions out of convenience; one can choose alternatives which may fit the empirical data closely or even use empirical data itself.

Table 3.3: Parameters for uniform renewable output distribution.

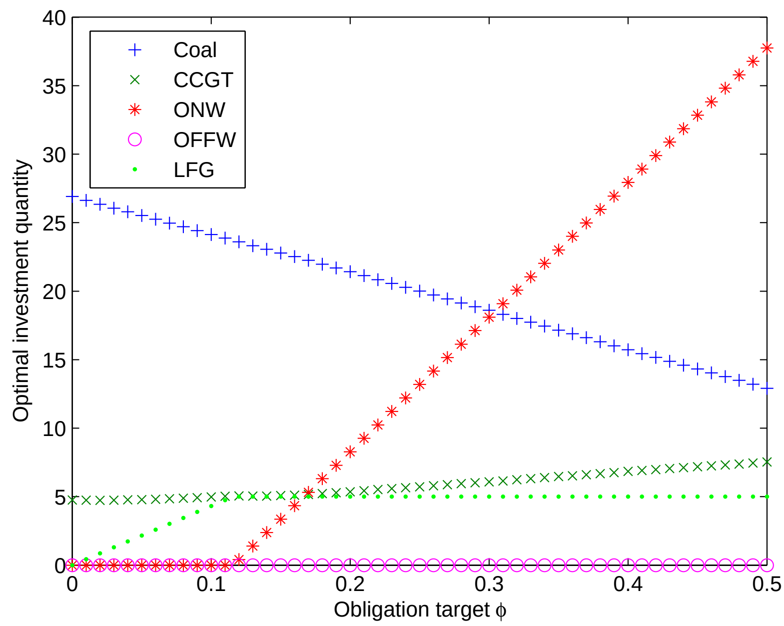
	a_k	b_k
ONW	.2	.36
OFFW	.33	.45
LFG	.5	.774

Finally, as investment in landfills in the UK is limited by law, we impose a maximum investment in LFG of 5 units.

3.6.2 Effects of Varying the Obligation Target

In this section we compare the three obligation systems in terms of investment quantities, prices, and CO_2 emissions for various obligation targets; that is, we let the obligation target range from 0 to 0.5 with increments of 0.01.

Figure 3.1: Investment quantities in the no banding system.



In Figures 3.1, 3.2, and 3.3 the effects of an increasing obligation target ϕ on investments are depicted. In Figure 3.1 the original obligation system without bandings is imposed. For low levels of ϕ we see that LFG is the only renewable technology in the mixture. As soon as it reaches its maximum capacity of 5 at $\phi = 0.12$, investments in ONW begin. There are no financial incentives to invest in OFFW (though in reality there may actually be investments in OFFW for other reasons than profit maximization, like geographic and regulatory constraints limiting onshore wind power). Investments in coal are decreasing with ϕ, but for the other conventional technology, CCGT, we observe a slight increase as ϕ increases. From our numerical output data we conclude that CCGT acts as a peak load technology, as it is mainly used for producing electricity in cases of high demand and/or low wind. More investment in ONW leads to more intermittency in the system. Consequently, to

Figure 3.2: Investment quantities in the UK banding system.

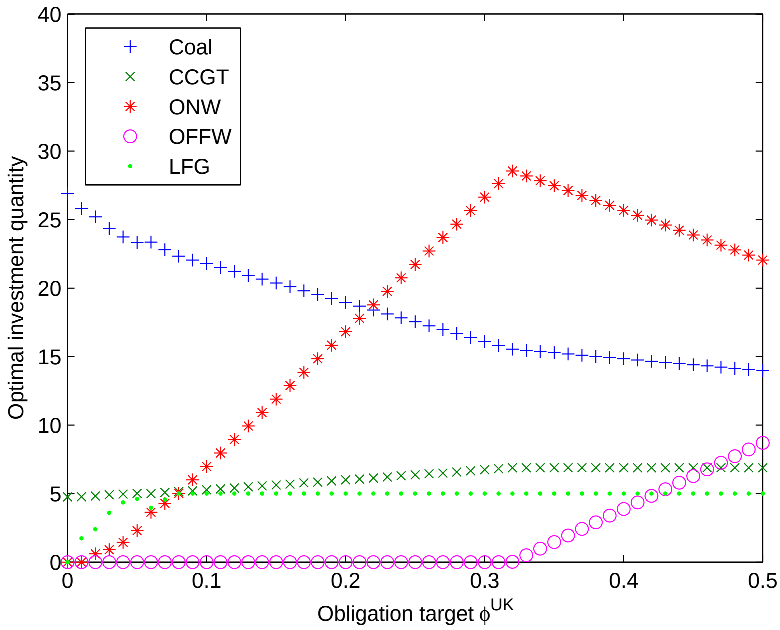
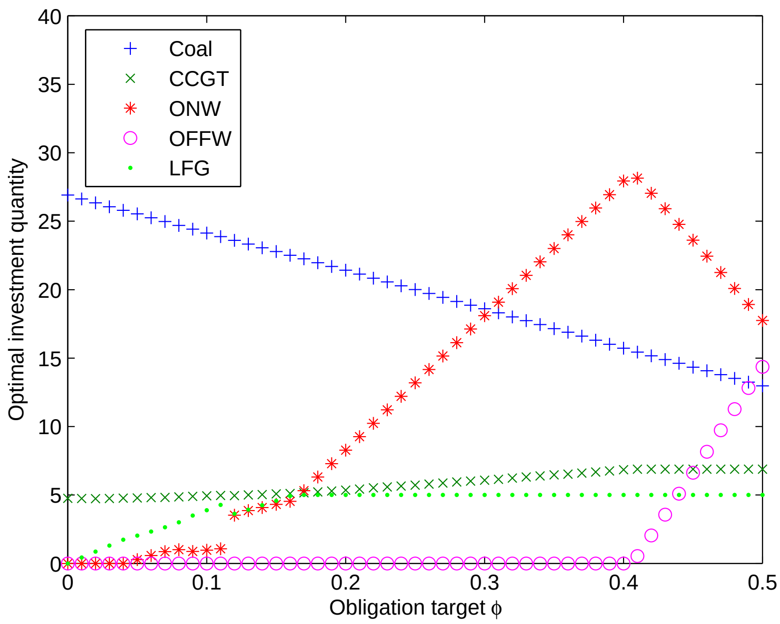


Figure 3.3: Investment quantities in the alternative banding system.

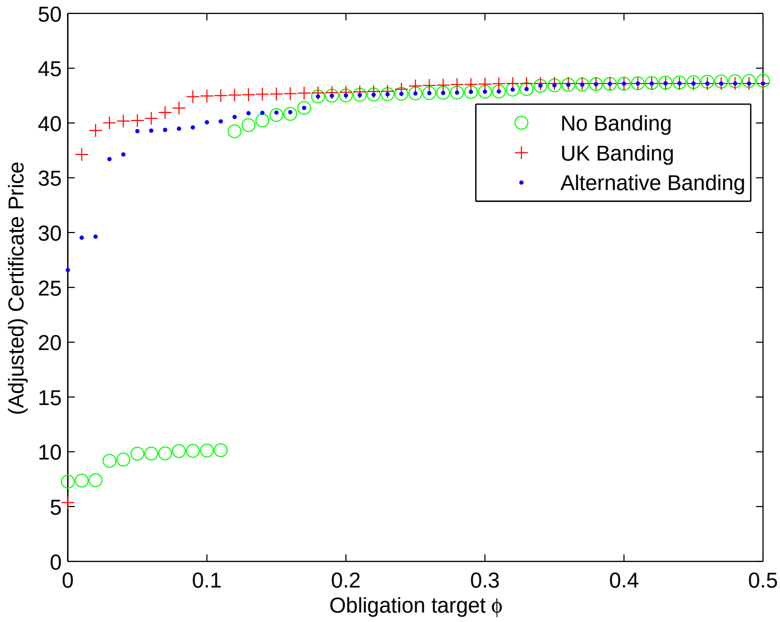


deal with the growing intermittency, more investment in the peak load technology, CCGT, will be done. Since investment in CCGT is cheaper than investment in coal, CCGT is preferred as a peak load technology. In the literature it is also often argued or even assumed that CCGT and gas in general are suitable technologies for dealing with intermittency due to its low capital cost and relatively fast start-up times (see, for example, DeCarolis and Keith (2006), Strbac (2002), Strbac et al. (2007)). Concluding, one can say that the obligation reduces CO_2 emissions in two ways, directly, via the replacement of coal by ONW, and indirectly, via the replacement of coal by the cleaner CCGT.

In Figure 3.2 investments under the UK banding system are depicted. We observe three major differences compared to Figure 3.1. First of all, for low levels of ϕ^{UK} there are no investments in ONW in the original obligation system (Figure 3.1). However, in the UK banding system both LFG and ONW are in the mixture. This is caused by the fact that firms have to satisfy a target expressed in certificates and LFG is only rewarded with 0.25 certificates per unit production. Secondly, we observe positive investments in OFFW for high levels of ϕ^{UK}. Starting from $\phi^{UK} = 0.33$, we see a trade off between ONW and OFFW investments. From the fact that OFFW is not in the optimal mixture until $\phi^{UK} = 0.33$, we can conclude that for the UK banding system to be effective in encouraging investments in OFFW one needs rather high target levels. In general one can conclude that the UK banding system is certainly not effective in giving incentives to invest in OFFW for all target levels. A third observation is that, as OFFW investments increase, there is no need for additional CCGT investments like we observed in Figure 3.1. This is due to the fact that OFFW is more reliable than ONW. As a consequence, investments in coal decrease less rapidly when OFFW is in the mixture.

Investments under the alternative banding system are depicted in Figure 3.3. For lower levels of ϕ we observe less investments in renewables than in the UK banding system, but about as much as in the original obligation system. A more obvious observation is that investment in OFFW begins at $\phi = 0.41$, which is much later than in the UK banding system (Figure 3.2). Hence, the alternative system needs higher obligation targets in order to succeed in giving financial incentives to invest in OFFW. The reason for this difference is the contribution of OFFW to satisfying the obligation target. In the UK system a unit production with OFFW is rewarded with 2 certificates and hence contributes to the target on certificates twice as much as ONW. In the alternative system a unit production with OFFW only contributes one unit to satisfying the target on renewable production. Hence, OFFW only enters the mixture when it has a cost advantage over ONW due to the higher bandings. In addition it appears that once OFFW has this cost advantage and is in the mixture, investments will increase more rapidly with ϕ compared to the UK banding system in which OFFW is mainly in the mixture to satisfy the target on certificates.

Figure 3.4: The certificate price ν in the no banding and UK banding systems, and the adjusted certificate price $\tilde{\nu}$ in the alternative banding system.



In Figure 3.4 the certificate prices are shown. For the original obligation system and the UK banding system we use the certificate price ν, while for the alternative banding system we use the adjusted certificate price $\tilde{\nu}$. Hence, what we observe in the figure is the rewards firms get per certificate. In general the certificate price is set in such a way as to make renewable technologies competitive compared to nonrenewable technologies (and eventually other renewables). We observe that without bandings and for ϕ up to 0.11, the certificate price can be low since LFG is relatively cheap. For higher ϕ and in the banding systems, ONW and OFFW are needed to satisfy the target and hence ν has to increase in order to make those technologies competitive. Since the production and investment costs in all three systems are the same, the reward needed for making a technology competitive is almost the same. Furthermore, it can be seen that the curves are relatively insensitive to changes in ϕ for $\phi > 0.11$. This can be explained by the fact that once the adequate level of ν is reached, there is no need for a higher reward.

Figure 3.5: The average consumer price.

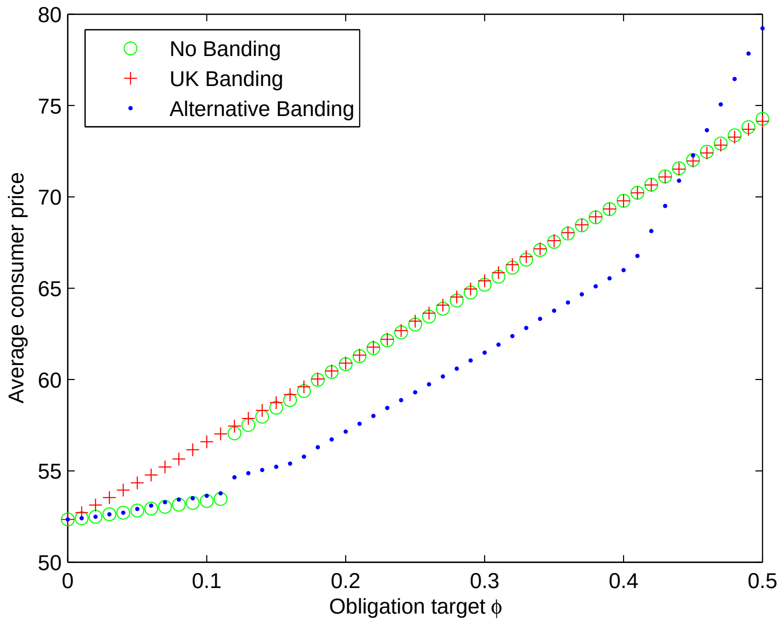
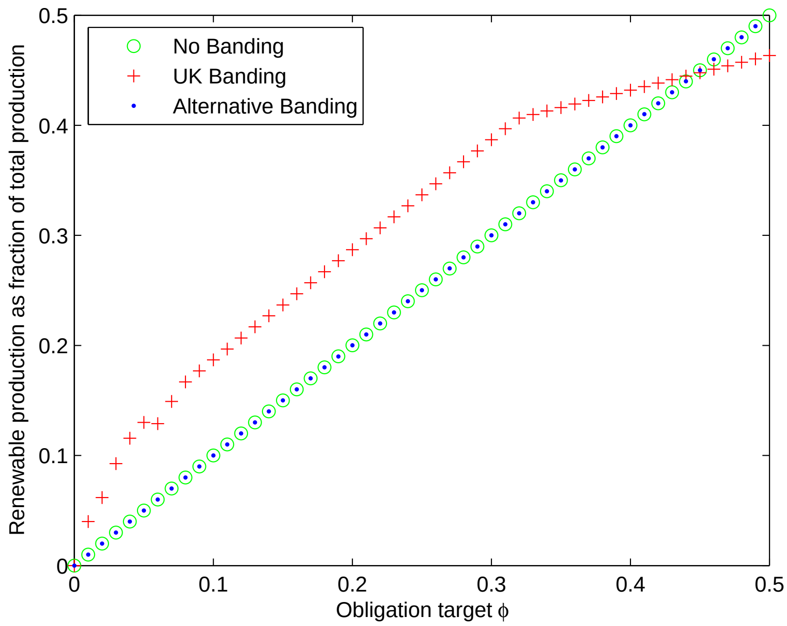


Figure 3.5 shows the average consumer prices. Since we have different prices in each realization we look at the average price, $\bar{p}^c$, over all realizations. As consumers pay $p^c = p^N + \phi\nu$ with ν relatively constant with respect to ϕ, as we saw in Figure 3.4, it is not surprising that in all three systems the average consumer price is increasing with ϕ. In the no banding system, the average price makes a jump when ϕ moves from 0.11 to 0.12, which is caused by the fact that ONW came into the mixture. In the UK banding system, there is no such jump, since ONW is in the optimal mixture also for low values of ϕ. From the figure we may conclude that going from the original obligation system to the UK banding system does not necessarily increase prices. Other than in the lower regions, the average prices in the no banding and UK banding systems are almost equal, meaning that the additional financial incentive given to OFFW in the form of a relatively high banding does not affect consumer prices. In the alternative banding system prices are in general lower than or equal to the prices in the other two systems, except for very high levels of ϕ ($\phi \geq 0.45$). Hence, in the alternative system the financial incentive given to OFFW does result in higher (average) consumer prices. This is caused by the fact that the reward per certificate will remain the same as we observed in Figure 3.4, while the total number of certificates significantly increases due to the larger amount of OFFW in the mixture. This is reflected in the consumer

prices.

Figure 3.6: The renewable production as a fraction of total production.

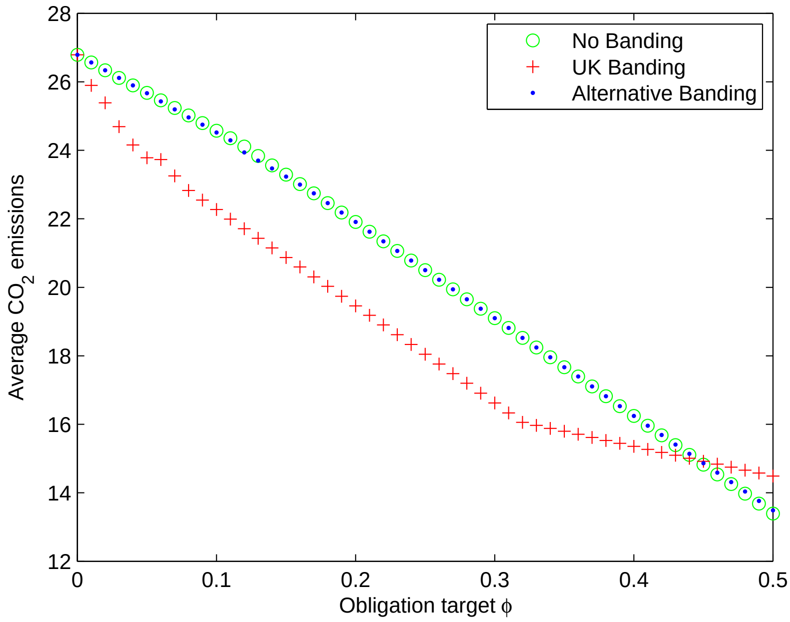


The realized renewable production as fractions of total production in the three scenarios are depicted in Figure 3.6. By the way the obligation was imposed in both the no banding and the alternative banding system the fraction will automatically be equal to ϕ, meaning that the target is actually satisfied. In the UK banding system, as long as only LFG and ONW are in the mixture, we are overshooting the target. However, for high levels of ϕ the original target on renewable production would be violated due to the increased amount of OFFW production. As a unit production with OFFW contributes two units to satisfying the target on certificates, less total renewable production is needed to satisfy this target.

In Figure 3.7 the average CO_2 emissions over all realizations are depicted. We see that, as one would expect, CO_2 emissions are decreasing with ϕ. Under the UK banding policy, for low and medium levels of ϕ there is less emission than in the other systems, because of the higher fraction of renewable production (as observed in Figure 3.6). This is mainly the case when OFFW is not in the optimal mixture. However, for higher levels of ϕ, when a significant amount of OFFW is in the mixture, we observe that the UK banding system leads to higher levels of CO_2 emission compared to the other systems.

To summarize, a renewable obligation leads to more financial incentives for investments in renewable resources. When OFFW is given more support in

Figure 3.7: The average CO_2 emissions.



the form of a higher banding, for relatively high obligation levels this support becomes effective. However, this may come at the cost of more CO_2 emissions in the UK banding system and significantly higher consumer prices in the alternative banding system.

3.6.3 Sensitivity to Decreasing Investment Costs

One goal of a banding system is to encourage development in less established technologies like offshore wind power. As technologies get more established, investment costs are likely to go down in the long run (learning by doing). We now assume that due to the extra support for offshore wind that is given by the banding systems, the investment cost of OFFW is going to decrease. In this section we will therefore analyze the effect of a small decrease in the unit investment cost of OFFW of 0.42 to $\kappa_4 = 53.06$. Assuming that operating and management costs remain the same, this means a decrease of 32.5 per KW of installed capacity (which is approximately 1% of the original building cost per KW).

In Figures 3.8 and 3.9 the investment quantities in the UK and alternative banding systems, respectively, are shown. Investment decisions in the original obligation system will be the same as in Figure 3.1. Comparing Figure 3.2 to Figure 3.8, it is obvious that a small decrease in the OFFW investment

Figure 3.8: Investment quantities in the UK banding system.

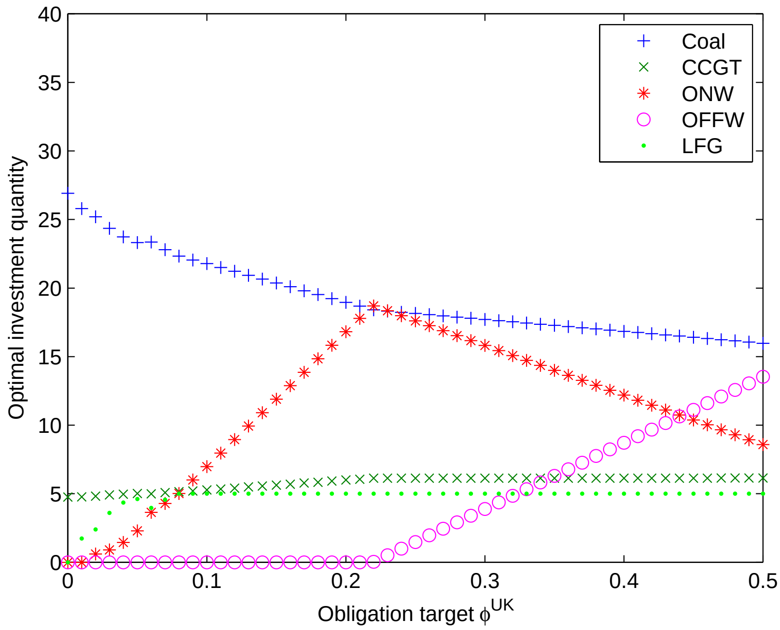
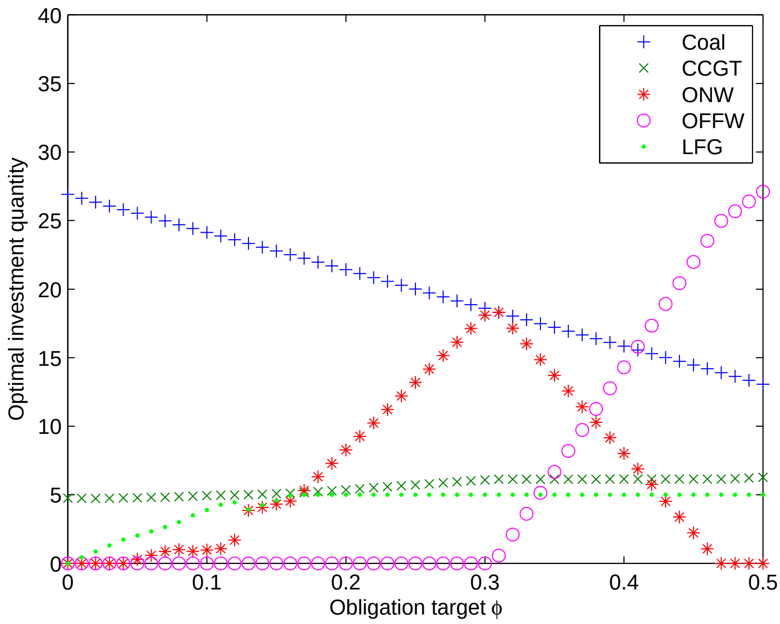


Figure 3.9: Investment quantities in the alternative banding system.



cost can have quite an impact on investment strategies. In Figure 3.2 a target of 0.33 is needed for OFFW to come into the mixture, whereas with the lower investment cost a ϕ of 0.22 is sufficient. The same comparison can be done for Figures 3.3 and 3.9. In Figure 3.3 a ϕ of 0.41 is needed to induce investments in OFFW, whereas in Figure 3.9 a ϕ of 0.31 is sufficient. Afterwards, investments in OFFW are increasing rapidly with ϕ, and ONW investments even go to zero. Comparing Figure 3.8 to Figure 3.9 we observe that again the UK banding system needs lower targets in order to induce investments in OFFW, but that for high levels of ϕ investments in OFFW are not increasing as rapidly as is the case in the alternative system.

Figure 3.10: The average consumer price.

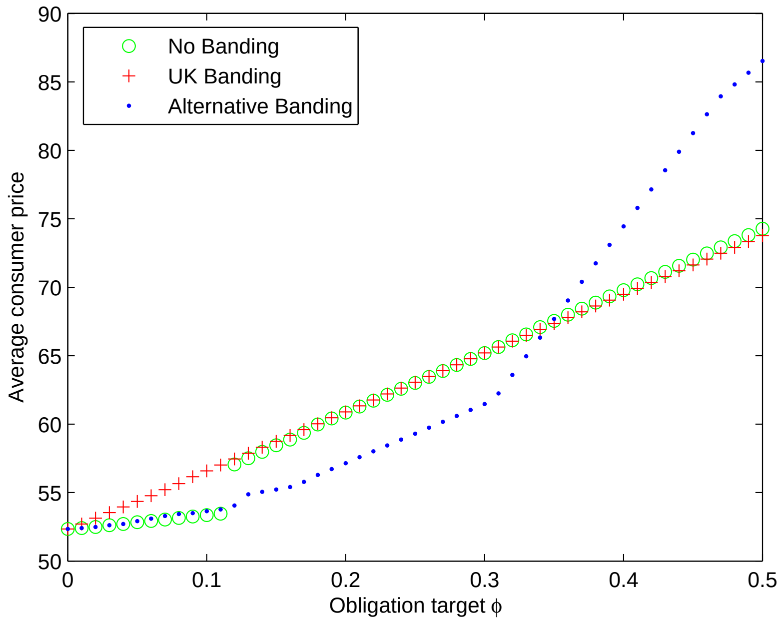
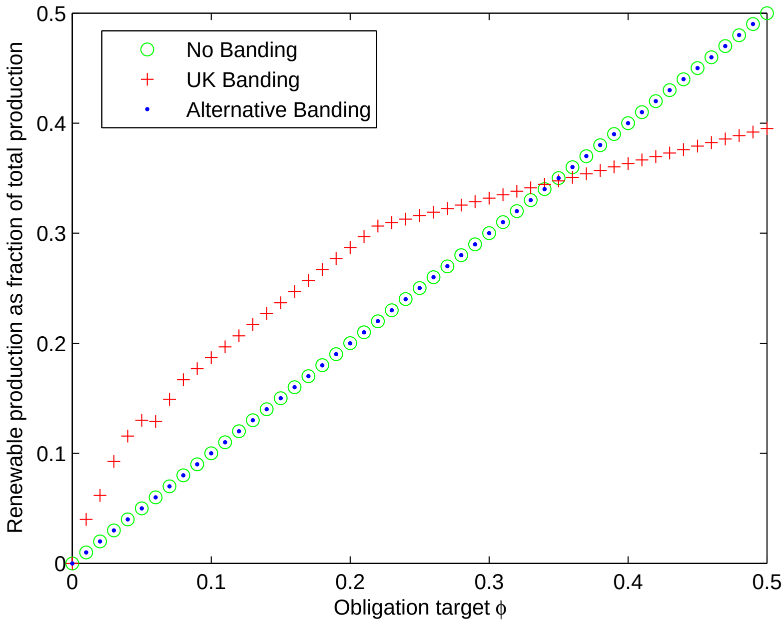


Figure 3.10 shows the average consumer prices after the cost reduction in OFFW. In the no banding and UK banding systems we observe nearly the same average prices as those observed in Figure 3.5. In the no banding system this is not surprising as OFFW is not in the mixture. In the UK banding system for high levels of ϕ there is more OFFW in the mixture than before the cost reduction. This additional OFFW does not influence the average consumer price. The consumer price consists of p^N, the price nonrenewable generators receive, which is independent of OFFW investment cost, plus $\phi\nu$, the mark-up. As ν is not influenced by the amount of OFFW in the mixture, the consumer price is not affected by a different mixture either. Obviously, in the alternative banding system OFFW does affect the average consumer price. Al-

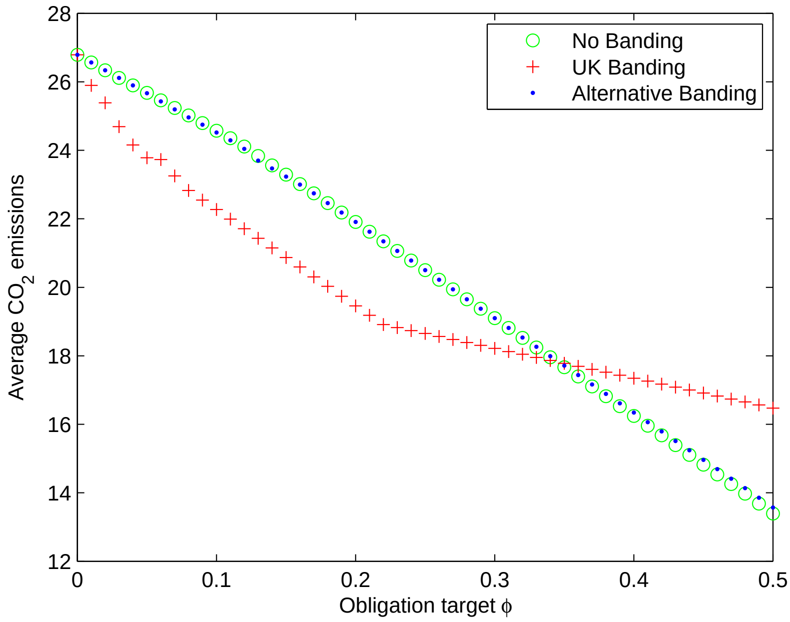
though the reward per certificate remains the same, the number of certificates significantly increases with the amount of OFFW investments. To cover the expenses of the increased number of certificates, consumer prices increase. We may conclude that in the alternative banding system consumers pay for the additional financial support that is given to OFFW. A reduction in OFFW investment cost can thus lead to a higher average consumer price.

Figure 3.11: The renewable production as a fraction of total production.



Figures 3.11 and 3.12 contain the fraction of renewable production and the expected CO_2 emissions, respectively. It can be seen that in the no banding and alternative banding system the obligation target is satisfied, and that CO_2 emissions are nearly the same. This was also observed in Figures 3.6 and 3.7. Hence, while prices are relatively stable in the UK banding system, emissions are relatively stable in the original obligation and alternative banding system. In the UK banding system, the fraction of renewable production and the expected CO_2 emissions are affected by the cost reduction. As can be seen from Figure 3.11, for $\phi \geq 0.35$ the obligation target is not satisfied, while it was satisfied up to $\phi = 0.44$ before the cost reduction. Also, when comparing Figures 3.7 and 3.12 we see that the cost reduction leads to more emissions for $\phi \geq 0.22$. The different mixture as a result of the OFFW cost reduction thus results in a more polluting system. Obviously this is a negative side effect of having the target on certificates, while OFFW receives 2 certificates per unit

Figure 3.12: The average CO_2 emissions.



production when only producing a single unit of renewable electricity. Calculation B could potentially improve the situation as it would put a higher target on certificates, but one should at least be cautious about the possible side effects of an obligation on certificates.

To summarize, when a technology with a high banding manages to reduce its cost, banding mechanisms may cause unwanted side effects. In the UK banding system a cost reduction in OFFW may lead to increased levels of CO_2 emission, whereas in the alternative system it may lead to increased consumer prices. A possible solution when OFFW costs are reduced, is reducing its banding coefficient accordingly.

3.7 Conclusions

This chapter modeled and analyzed three different renewable obligation policies in a mathematical framework. The original (UK) renewable obligation, the renewable obligation including the UK banding system, and a proposal for an alternative banding system have been modeled in the electricity market investment model that was originally introduced by Gürkan et al. (2013). Then a numerical study shed some light on the possible advantages and disadvantages of each system and we have observed a couple of side effects.

The main goal of the renewable obligation is to reduce CO_2 emissions by reducing investments in polluting technologies and to have them replaced by renewable ones. As we observed in our numerical study, CO_2 emissions are curbed both directly by the replacement of coal by onshore wind, and indirectly by the replacement of coal by the cleaner CCGT. While we see an increasing investment in onshore wind and to a lesser extent landfill gas, a major concern is that in the long term targets may not be met without investment in less developed technologies like offshore wind. As can also be observed in our numerical study, the renewable obligation fails to give the right financial incentives to these less developed technologies. This is the main reason why a banding system was introduced in the UK in 2009. Rather than rewarding each unit production with a renewable resource with the same amount of certificates, more certificates and hence more support is given to the technologies that needed additional incentives.

We find that the banding system can be successful in giving incentives to OFFW, but that rather high targets are needed in order to do so. Once there is a substantial amount of investment in OFFW, we observe that the original obligation target on production may not be met. This is caused by the fact that a unit production with OFFW adds two units to satisfying the obligation target whereas it only adds one unit of renewable energy. This problem may be magnified when less developed technologies like tidal and wave power enter the optimal mixture. They need more support as stressed out by Allan et al. (2011), and therefore they have banding coefficients of 3 and 5, respectively.

We then proposed an alternative banding system, which guarantees that the original obligation target on production is always satisfied. On the downside, based on our numerical study it is expected that in the alternative system prices will significantly rise and that even higher targets are needed in order to give incentives for investment in tidal and wave power.

It is expected that more support for OFFW is going to cause more development in OFFW and hence will result in a downward shift in OFFW investment cost in the long run. We analyzed the consequences of such a cost decrease and found that investment levels in different technologies are in general very sensitive. Since more OFFW comes into the optimal technology mixture, less investment in renewable technologies is needed to satisfy the obligation in the UK banding system. Therefore we find that a cost decrease in OFFW actually results in more CO_2 emissions. We do not observe this in the alternative banding system, but there the prices are very sensitive to changes in cost and we find a significant increase in the consumer price. Obviously, bandings, besides its positive effects on less established technologies, can have certain negative side effects and can give the wrong message in the long run. When technologies succeed in reducing its cost as a result of the given support (learn by doing), the financial support should be reduced accordingly.

CHAPTER 4

PRICES VERSUS QUANTITIES: CAN RENEWABLE ENERGY QUOTA BE ACHIEVED UNDER FIXED FEED-IN TARIFF POLICIES?

4.1 Introduction

Governments aim at meeting certain quotas on the amount of renewable electricity production in electricity markets. As investment in renewable resources is costly, without financial support, firms are typically inclined to invest in non-renewable resources like coal and gas plants. Therefore, in order to compensate for the high investment and to stimulate development in existing and new renewable resources, financial incentives must be given. A regulator has two ways of giving these incentives, namely via quantity based and via price based instruments.

In a quantity based renewable energy policy, the regulator sets a minimum quantity or percentage of electricity production that must come from renewable resources. This target or quota is typically imposed as an obligation on electricity producers or consumers, and is set for a certain period, typically one year or a couple of years. In order to show compliance to the target, tradable green certificates are handed out for each unit production with a renewable resource. These certificates are tradable in a secondary market and have a certain monetary value. This value, which is referred to as the certificate price, is determined by the market. More specifically, the certificate price is determined by the demand for certificates and the fine firms pay when not meeting the target set by the regulator. The certificate price adds to the short term revenue of the renewable electricity producers and hence helps in covering their high investment cost. By creating a market for certificates, the regulator is giving producers an indirect subsidy. Quantity based instruments are used in for example UK, Italy, Belgium, Romania, and Poland.

When price based instruments are used, instead of fixing the quantities, prices are fixed. The regulator regulates the prices renewable electricity producers get for a unit production with a renewable resource. These prices, referred to as feed-in tariffs (FITs), can be dependent or independent of the price of electricity and ultimately depend on the policy imposed by the regulator. The level of the tariff is typically based on effective marginal or so-called levelised cost of generation, that is, the costs for a unit production, including investment, fuel, operations and management, and other costs. Since effective marginal costs differ among renewable technologies, feed-in tariffs are often differentiated based on technology. In some cases, tariffs are also differentiated based on other factors like time of the year (Czech Republic, Hungary, and Portugal), installed capacity (France, Germany, Luxembourg, Slovenia, and Spain), overall electricity generation (Austria), and grid connection (Greek Islands), see Klein (2008). Since feed-in tariffs are in general paid on top of or instead of the price of electricity, the feed-in tariffs are a direct subsidy. These subsidies are paid either by tax payers or by electricity consumers (ratepayers). In this chapter we deal with the fixed feed-in policy that pays firms a fixed price instead of the electricity price, and we assume that the tariffs are paid by the tax payers.

In the literature, many comparisons between quantity based certificate systems and price based feed-in tariff systems are made. Feed-in tariffs are generally considered as favorable due to the lower market risk as a result of long-term price contracts, see for example Butler and Neuhoff (2008) and Mitchell et al. (2006). The latter also concludes that feed-in tariff systems have led to higher levels of renewable investments in practice compared to certificate systems. Furthermore, a feed-in system is capable of promoting different types of technologies by handing out differentiated tariffs whereas quantity based systems single out the least costly renewable technology as argued in del Rio and Gual (2007). Broad mixtures can be desirable as investments in new and less established technologies can lead to research and development in these technologies, resulting in more efficient use and lower operating costs in the long term.

One specific theoretical drawback of FITs is the following. Suppose there is a choice between investing in a non-renewable and a renewable technology. Typically, without any government regulation there is no incentive to invest in the renewable technology due to the high effective marginal costs as a result of high investment costs and the random availability of capacity. It is pointed out in the literature that when the difference between the effective marginal costs of both technologies is constant, price based instruments can be seen as inferior to quantity based instruments as quota cannot be guaranteed under a price based policy. Such a situation is analyzed in Requate and Unold (2003) for taxation versus emission permits in a market with a polluting and an abatement (clean) technology. Under permits, an optimal permit price will be set by the market and quota will be met. Under taxation, for a tax level higher than the optimal permit price, all firms invest in the clean technology. For a tax level lower than the optimal permit price, no firm will invest in the clean technology. When the tax level is set equal to the optimal permit price, firms are indifferent between both technologies. This implies that in a decentralized market, the price based instrument alone will not necessarily result in quotas being met. A similar example can be found in Chapter 2, in which we showed that when the optimal permit price is set as a tax, multiple equilibria exist; only one of these equilibria satisfies the emission cap.

Since certificates are similar to permits and FITs are similar to taxes, the same issue is expected in case we compare FITs to certificates. We show that indeed, under the assumption of constant effective marginal cost and hence a constant difference in marginal cost between a non-renewable and renewable technology, meeting renewable quota cannot be guaranteed under an FIT policy. In order to show this, we consider a two-stage investment model for a perfectly competitive electricity market, similar to the models used in Chapter 2 and Chapter 3. At the first stage, firms simultaneously decide on their investment quantities as to maximize their long term profits. At the second stage, firms decide on their production quantities, random demand realizations and random availability of renewable capacity are revealed, and the market is cleared. We impose a renewable electricity obligation on the first stage of the problem, employ this to derive the optimal certificate price, and use this information as a benchmark for determining the best possible feed-in tariff. We then show that in the absence of an explicit obligation, only a price based FIT cannot guarantee that quota are met.

Contrary to those analytical results, in reality FIT systems seem to perform well as mentioned earlier. The targets set by regulators are often met without explicitly imposing them on firms. For this practical reason, we question whether the constant effective marginal cost assumption is a realistic one. Furthermore, as argued by Weitzman (1974), any outcome that can be achieved under quantity based policies can be achieved by means of a price based policy, as long as convexity assumptions hold. Hence, as under linear investment cost assumptions a FIT system seems to be inferior to a quota system, it is natural to move to non-linear convex investment costs for renewable technologies. Assuming non-linear convex investment cost functions simply means that with each additional unit of capacity installed, installing this unit is more costly than the previous unit. Convex cost assumptions in electricity markets are not uncommon in the literature, see for example Reichenbach and Requate (2012) and Traber and Kemfert (2011). However, to the best of our knowledge there is no literature studying the consequences of different cost assumptions on feed-in policies, in particular in the presence of uncertainty. One could also study non-linear concave cost functions, but this is beyond the scope of this chapter.

We show that under non-linear convex investment costs in renewable resources, FITs can indeed achieve the same outcomes as a certificate system. In particular, we impose an alternative obligation type constraint on the market to obtain a benchmark for the optimal feed-in tariffs. Then we remove the explicit obligation and show that with the feed-in tariffs obtained the obligation is still satisfied. Moreover, when multiple renewable technologies are available, a regulator has the freedom to set the tariff levels in such a way that any renewable technology can be in the optimal technology mixture. We consider parameter vectors determining the feed-in tariffs for each technology and aim to find for each possible parameter vector the corresponding technology mixture at the market equilibrium. We do this by finding vectors for which a single technology is invested in and for which a slight deviation results in another technology entering the mixture. This is then used to divide the space of all possible parameter choices into areas where one, two, or three renewable technologies are in the mixture. Finally, for the case of more than three renewable technologies, we find the feed-in parameters for which one technology is singled out.

In a numerical study, we consider a small market with two non-renewable technologies and three renewable technologies, and provide a numerical application of the analytical results derived in this chapter. In particular, we consider the effects of different choices of the parameters determining the feed-in tariffs, as set by the regulator, on the optimal investment mixture. We show that in case of linear investment cost functions typically one technology is singled out regardless of the feed-in parameters set by the regulator. Furthermore, in the absence of an explicit obligation, meeting a renewable quota cannot be guaranteed. When investment cost functions are quadratic we find for each possible parameter vector the exact technology mixture. Furthermore, we find areas for which two and three renewable technologies are in the optimal mixture. A regulator can thus choose a feed-in parameter vector based on the desired mixture using our theory. We show that removing the obligation that is used to obtain a benchmark value for the optimal feed-in tariff does not change the optimal technology mixture. We also consider two cases where renewable technologies have non-linear non-quadratic convex cost functions. While analytically we cannot determine areas in which two or three technologies are in the optimal mixture, our numerical tools provide a way to determine these by running a series of numerical experiments. Finally, our observations lead to a theoretical result for two specific classes of cost functions, namely functions with a superadditive non-constant part and functions with a subadditive non-constant part; when investment cost functions are of either form, the areas in which two or three technologies are in the optimal mixture are characterized by a specific shape.

This chapter is structured as follows. In Section 4.2, we present the electricity market investment model. In Section 4.3, we introduce the renewable obligation and the concept of a certificate system. In Section 4.4, we explain the details of the feed-in tariff system. Our main analysis, theorems, and results are presented in Section 4.5, in which we compare a system with linear investment cost to a system with non-linear convex investment cost. In Section 4.6, we carry out our numerical study. Section 4.7 includes the conclusions.

4.2 The Electricity Market Investment Model

We present a mathematical model for an electricity market with random demand and uncertain output of renewable resources. We first describe the electricity market and its characteristics and state the assumptions we make. Then we present the model and do some preliminary analysis.

The electricity market is represented by an electricity grid with supply and demand nodes. A set of transmission lines connects the nodes and forms the electricity network. Transmission lines are usually owned by a transmission system operator (TSO). At supply nodes, firms owning production capacity produce electricity. Their production capacity consists of non-renewable and renewable technologies from which electricity can be generated. Outputs of renewable resources like wind and solar power are typically dependent on weather conditions and therefore uncertain. Production capacities can be expanded when firms invest in additional capacity, which is typically done on a monthly or yearly basis. At demand nodes, consumers with an hourly or daily random, and by assumption inelastic, demand are located. Each period electricity, which is a non-storable good, is transmitted from supply nodes to demand nodes, using the transmission lines in the network, in order to satisfy demand. Transmission from one node to another may affect flows in the entire network.

Decisions by firms and the TSO are made in two stages. At the first stage, long-term decisions are made. All firms, at the beginning of the year or month, simultaneously maximize their long-term profits while deciding on their investment in (additional) production capacity. Long-term profits also depend on daily or hourly outcomes that are unknown at the time of making the decision. These daily or hourly outcomes result from the second stage, which can be seen as a short-term process that repeats itself every day or hour and is referred to as the spot market. In the spot market, firms produce power in order to maximize their short term profits, the TSO decides on the flows through the transmission lines while ensuring reliability of the system and maximizing its own profits from buying electricity at supply nodes and selling it to consumers at demand nodes. Spot market prices are determined by the market, which goes as follows. Given the first stage investment quantities, an equilibrium to the second stage between all firms and the TSO is determined. At an equilibrium, the price in each node is set such that supply meets demand, unless there is unsatisfied demand. When that is the case, the price in the corresponding node will be set equal to an imposed price cap, the value of lost load, which in the literature is referred to as VOLL pricing; see for example Stoft (2002) and Ehrenmann and Smeers (2008). Demand and available generation capacity in renewable resources are assumed to be unknown at the first stage. Firms only know the underlying probability distributions (potentially based on past empirical data) of demand and generation output, while their realizations will be revealed at the second stage. We assume perfect competition at both stages, meaning that none of the operators in the market is aware of the fact that by behaving strategically, they can influence the market price. As decisions at both stages (indirectly) depend on decisions made by all firms at both stages and by the TSO at the second stage, the two-stage problem can be seen as a two-stage game between firms and the TSO. An equilibrium to this game is found when for none of the operators it is profitable to deviate.

We present a mathematical representation of the electricity market as a two-stage game between firms and the TSO under perfect competition, similar to the models presented in Chapter 2 and Chapter 3. We present a stochastic version the model, excluding a renewable obligation or feed-in tariffs; these will be added later. In the model, each firm is assumed to have its own unique technology. Therefore, the terms firm and technology will be used interchangeably. Furthermore, each firm is assumed to be operational at all supply nodes (but not necessarily producing). Sets, variables, and parameters used in the model are given below.

Sets:

N	:	set of demand nodes
I	:	set of supply nodes
K^N	:	set of non-renewable technologies
K^R	:	set of renewable technologies
K	:	set of all technologies $(K := K^N \cup K^R)$
L	:	set of electricity transmission lines connecting nodes in the network.

Parameters:

c_{ik}	:	unit production cost at supply node $i \in I$ for technology $k \in K$
$\kappa_{ik}(x_{ik})$	:	investment cost as a function of x_{ik} at supply node $i \in I$ for technology $k \in K$
$PTDF_{l,j}$	:	power transmitted through line $l \in L$ due to one unit of power injection into node $j \in N \cup I$
h_l	:	capacity limit of line $l \in L$
$VOLL$	:	value of unserved energy or lost load.

Variables:

x_{ik}	:	generation capacity investment in technology $k \in K$ at supply node $i \in I$
y_{ik}	:	quantity of power generated at supply node $i \in I$ by using technology $k \in K$
f_j	:	net power flow dispatched by the TSO to node $j \in N \cup I$
δ_j	:	unserved demand at node $j \in N \cup I$
p^e_j	:	electricity price at node $j \in N \cup I$
p^N_{ik}	:	price non-renewable technology $k \in K^N$ at supply node $i \in I$ gets per unit sold
p^R_{ik}	:	price renewable technology $k \in K^R$ at supply node $i \in I$ gets per unit sold.

Uncertainty is dealt with as follows. Let $\omega \in \Omega$ be a random vector in the space of possible outcomes Ω. The vector ω contains outcomes for both random demand and random available capacity of renewable resources, which have some joint distribution Ψ. Random available capacity for renewable technology $k \in K^R$ in supply node $i \in I$ is denoted by $F_{ik}(x, \omega)$, some nondecreasing differentiable possibly nonlinear random function of investment quantity x. The function $F(\cdot)$ reflects that for renewable technologies not all capacity is available for production at all times, due to its dependence on for example wheather conditions. Random demand in demand node $n \in N$ is denoted by $d_n(\omega)$.

In the remainder of this chapter, variables may get superscripts N and R depending on their corresponding technology in K^N and K^R, respectively. In addition, for $k \in K$ we define $x_k = (x_{ik})_{i \in I}$ and $y_k = (y_{ik})_{i \in I}$, the vectors containing investments and production, respectively, of technology $k \in K$ in all supply nodes.

At the first stage, each firm $k \in K$ simultaneously maximizes its long-term profit while deciding on its optimal investment quantities x_k in all supply nodes. A firm's long-term profit consists of its expected second stage profit minus its total investment cost. The investment cost for technology $k \in K$ in supply node $i \in I$ is a function of the investment quantity x_{ik} and is denoted by $\kappa_{ik}(x_{ik})$. Throughout the chapter we assume linear investment cost for each non-renewable technology $k \in K^N$ in supply node $i \in I$ and thus write $\kappa^N_{ik}(x^N_{ik}) = \kappa^N_{ik} x^N_{ik}$. For renewable technologies we assume the investment cost functions to be nonnegative and differentiable; we specify the functions later. The second stage profit in a realization $\omega \in \Omega$ for technology $k \in K$ in supply node $i \in I$ consists of the price $p_{ik}(\omega)$ paid for a unit production, minus marginal production cost c_{ik}, times production quantity $y_{ik}(x_k, \omega)$ as a function of investment quantities x_k. The expectation with respect to ω gives the expected second stage profit. For non-renewable technology $k \in K^N$ the first stage problem is:

$$\max_{x^N_k \geq 0} \quad E_\omega \left[\sum_{i \in I} (p^N_{ik}(\omega) - c^N_{ik}) y^N_{ik}(x^N_k, \omega) \right] - \sum_{i \in I} \kappa^N_{ik} x^N_{ik}. \tag{4.1}$$

Here, $y^N_{ik}(x^N_k, \omega)$ is the optimal production quantity of non-renewable technology $k \in K^N$ in supply node $i \in I$ in realization $\omega \in \Omega$ at the second stage if x^N_k is the investment quantity. The price $p^N_{ik}(\omega)$ paid per unit production with technology $k \in K^N$ in supply node $i \in I$ is taken from the second stage and dealt with as a parameter in (4.1), since we assume perfect competition and hence that firms are price takers. Typically, non-renewable technologies receive the price of electricity, defined as $p_i^e(\omega)$ in supply node $i \in I$ per unit production, but depending on the pricing scheme imposed by a regulator this may be altered.

For renewable technology $k \in K^R$ the first stage problem is similar to (4.1), except from the fact that investments in a renewable technology may be limited either by law or by physical limitations such as a lack of space or resources. For example, in the UK electricity market, investments in landfill gas (LFG) are capped by law (by means of permits). As such, we introduce for each renewable technology $k \in K^R$ a maximum investment quantity M_k^R, which can be infinite for certain technologies. The first stage problem for renewable technology $k \in K^R$ is:

$$\begin{aligned} \max_{x_k^R \geq 0} \quad & E_\omega \left[\sum_{i \in I} (p_{ik}^R(\omega) - c_{ik}^R) y_{ik}^R(x_k^R, \omega) \right] - \sum_{i \in I} \kappa_{ik}^R(x_{ik}^R) \\ \text{s.t.} \quad & \sum_{i \in I} x_{ik}^R \leq M_k^R \qquad\qquad (\zeta_k^R). \end{aligned} \tag{4.2}$$

Here, ζ_k^R is the dual variable with respect to the ceiling on investments in technology $k \in K^R$ and represents the scarcity rent for additional resources. Again, $y_{ik}^R(x_k^R, \omega)$ is the optimal production quantity of renewable technology $k \in K^R$ in supply node $i \in I$ in realization $\omega \in \Omega$ at the second stage if x_k^R is the investment quantity. The price $p_{ik}^R(\omega)$ is taken from the second stage and dealt with as a parameter. Without any regulation to support renewable technologies, this price would equal the electricity price $p_i^e(\omega)$ in supply node $i \in I$. However, depending on regulation and subsidies this may be altered via a pricing scheme imposed by a regulator.

At the second stage, in each realization of the random demand and random generation output of renewable resources, for given first-stage decisions, firms and the TSO make decisions while the market is cleared. More specifically, in each realization $\omega \in \Omega$ and for given x_k^N and $p_{ik}^N(\omega)$ for all $i \in I$, non-renewable firm $k \in K^N$ maximizes its short-term profit while determining its optimal production quantity $y_k^N(x_k^N, \omega)$ in all its supply nodes by solving

$$\begin{aligned} \Pi_k^N(x_k^N, \omega) := \max_{y_k^N(\omega)} \quad & \sum_{i \in I} (p_{ik}^N(\omega) - c_{ik}^N) y_{ik}^N(\omega) \\ \text{s.t.} \quad & y_{ik}^N(\omega) \leq x_{ik}^N \qquad (\beta_{ik}^N(\omega)) \qquad \forall i \in I \\ & y_{ik}^N(\omega) \geq 0 \qquad\qquad\qquad \forall i \in I. \end{aligned} \tag{4.3}$$

Here, $\beta_{ik}^N(\omega)$, $k \in K^N$, $i \in I$, is the dual variable associated with the capacity constraint. It represents the scarcity rent of capacity in non-renewable technology $k \in K^N$ in supply node $i \in I$ in realization $\omega \in \Omega$. The price $p_{ik}^N(\omega)$ for technology $k \in K^N$ in supply node $i \in I$ in realization $\omega \in \Omega$ is taken as a parameter. Different demand realizations may result in different prices at equilibrium, and hence the price is dependent on $\omega \in \Omega$ and is determined at the second stage equilibrium as we discuss below.

Similarly, renewable technology $k \in K^R$ maximizes, in each realization $\omega \in \Omega$ and for given x_k^R, its short-term profit while determining its optimal production quantity $y_k^R(x_k^R, \omega)$ in all its supply nodes by solving

$$\begin{aligned} \Pi_k^R(x_k^R, \omega) := \max_{y_k^R(\omega)} \quad & \sum_{i \in I} (p_{ik}^R(\omega) - c_{ik}^R) y_{ik}^R(\omega) \\ \text{s.t.} \quad & y_{ik}^R(\omega) \leq F_{ik}(x_{ik}^R, \omega) \qquad (\beta_{ik}^R(\omega)) \qquad \forall i \in I \\ & y_{ik}^R(\omega) \geq 0 \qquad \forall i \in I. \end{aligned} \tag{4.4}$$

Again, $\beta_{ik}^R(\omega)$, $k \in K^R$, $i \in I$, is the dual variable associated with the capacity constraint. It represents the scarcity rent of available capacity in renewable technology $k \in K^R$ in supply node $i \in I$ in realization $\omega \in \Omega$.

A transmission system operator (TSO) optimizes its short-term profits from buying electricity from supply nodes, and transmitting and selling it to demand nodes. The TSO is a price-taker and thus takes in each realization $\omega \in \Omega$ electricity prices $p_j^e(\omega)$ in each node $j \in N \cup I$ as given. Its decision variables are the flows, f_j, into or out of each node $j \in N \cup I$. $f_j > 0$ indicates a flow into node j, whereas $f_j < 0$ indicates a flow out of node j. Electricity is transmitted through the network of transmission lines $l \in L$ that run from one node to another. Typically, a power injection into one node, affects flows on all transmission lines. These effects can be positive or negative, and are given by so-called power transmission distribution factors (PTDF), which are based on Kirchhoff's Law, see for more details Chao et al. (2000). On each transmission line $l \in L$ there can be a capacity h_l limiting the net load in both directions, that is, the net flow on line l has to be between $-h_l$ and h_l. The TSO determines in each realization $\omega \in \Omega$ the optimal flows $f^*(\omega)$ in all nodes by solving

$$\begin{aligned} \max_{f(\omega)} \quad & \sum_{j \in N \cup I} p^e_j(\omega) f_j(\omega) \\ \text{s.t.} \quad & \sum_{j \in N \cup I} f_j(\omega) = 0 && (\rho(\omega)) \\ & h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j(\omega) \geq 0 && (\lambda^+_l(\omega)) \quad \forall l \in L \\ & h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j(\omega) \geq 0 && (\lambda^-_l(\omega)) \quad \forall l \in L. \end{aligned} \tag{4.5}$$

The first constraint balances the flows such that the total flow out of all nodes equals the total flow into all nodes, with (free) dual variable ρ. The second and third constraint make sure that the net flow on each line $l \in L$ remains between its limits h_l and $-h_l$, with dual variables λ^+_l and λ^-_l, respectively.

In each realization $\omega \in \Omega$, the market is cleared by means of two types of market clearing conditions. The first type determines in each realization $\omega \in \Omega$ the electricity price $p^e_j(\omega)$ in each node $j \in N \cup I$ while balancing supply and demand. In each supply node $i \in I$, where demand in each $\omega \in \Omega$ is defined as $d_i(\omega) = 0$, the flow out of the node can be at most the total production in the node. In each demand node $n \in N$, where production in each $\omega \in \Omega$ is defined as $y^N_{nk}(\omega) = 0$, $k \in K^N$, and $y^R_{nk}(\omega) = 0$, $k \in K^R$, the flow into the node should be at least the demand, unless there is unsatisfied demand (see below). Hence, for all nodes $j \in N \cup I$ we have a constraint taking care of supply and demand. The price of electricity in each node is determined perpendicular to each of these constraints. The second type deals with unsatisfied demand. When in $\omega \in \Omega$ there is unsatisfied demand in node $j \in N \cup I$, which we define as $\delta_j(\omega)$, then the price in that node will be set at $VOLL$, the value of lost load. This is an, in general, relatively high number that serves as a price cap in case of demand curtailment. The market clearing conditions imposed on the second stage problem in each realization $\omega \in \Omega$ are the following:

$$\begin{aligned} 0 \leq \sum_{k \in K^N} y^N_{jk}(\omega) + \sum_{k \in K^R} y^R_{jk}(\omega) + \delta_j(\omega) + & \\ f_j(\omega) - d_j(\omega) & \perp p^e_j(\omega) \geq 0 \quad \forall j \in N \cup I \\ 0 \leq VOLL - p^e_j(\omega) & \perp \delta_j(\omega) \geq 0 \quad \forall j \in N \cup I. \end{aligned} \tag{4.6}$$

Summarizing, the second stage in each realization $\omega \in \Omega$ consists of optimization problem (4.3) for all non-renewable technologies $k \in K^N$, optimization problem (4.4) for all renewable technologies $k \in K^R$, the TSO's problem (4.5), and the market clearing conditions (4.6). Given a pricing scheme that expresses in each $\omega \in \Omega$ the prices $p_{ik}^N(\omega)$, $k \in K^N$, $i \in I$, and $p_{ik}^R(\omega)$, $k \in K^R$, $i \in I$, in terms of the price of electricity $p_i^e(\omega)$ in supply node $i \in I$, then the entire second stage can be solved by means of a mixed complementarity problem (MCP) consisting of the KKT optimality conditions to all the second stage problems. Given x, in any $\omega \in \Omega$ the MCP that solves the second stage to optimality is to find $y^*(\omega), \delta^*(\omega), p^{e*}(\omega), \beta^*(\omega), \lambda^{*+}(\omega), \lambda^{*-}(\omega), \rho^*(\omega), f^*(\omega)$ that satisfy the following set of KKT-conditions:

$$
\begin{array}{rcll}
0 \leq \beta_{ik}^{*N}(\omega) - p_{ik}^{*N}(\omega) + c_{ik}^N & \perp & y_{ik}^{*N}(\omega) \geq 0 & \forall i \in I, k \in K^N \\
0 \leq \beta_{ik}^{*R}(\omega) - p_{ik}^{*R}(\omega) + c_{ik}^R & \perp & y_{ik}^{*R}(\omega) \geq 0 & \forall i \in I, k \in K^R \\
0 \leq x_{ik}^N - y_{ik}^{*N}(\omega) & \perp & \beta_{ik}^{*N}(\omega) \geq 0 & \forall i \in I, k \in K^N \\
0 \leq F_{ik}(x_{ik}^R, \omega) - y_{ik}^{*R}(\omega) & \perp & \beta_{ik}^{*R}(\omega) \geq 0 & \forall i \in I, k \in K^R \\
0 \leq VOLL - p_j^{*e}(\omega) & \perp & \delta_j^*(\omega) \geq 0 & \forall j \in N \cup I \\
0 \leq \sum_{k \in K^N} y_{jk}^{*N}(\omega) + \sum_{k \in K^R} y_{jk}^{*R}(\omega) + & & & \\
\delta_j^*(\omega) + f_j^*(\omega) - d_j(\omega) & \perp & p_j^{*e}(\omega) \geq 0 & \forall j \in N \cup I \\
0 \leq h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j^*(\omega) & \perp & \lambda_l^{*+}(\omega) \geq 0 & \forall l \in L \\
0 \leq h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j^*(\omega) & \perp & \lambda_l^{*-}(\omega) \geq 0 & \forall l \in L \\
\sum_{l \in L} PTDF_{l,j}(\lambda_l^{*+}(\omega) - \lambda_l^{*-}(\omega)) + & & & \\
p_j^{*e}(\omega) - \rho^*(\omega) = 0 & & & \forall j \in N \cup I \\
\sum_{j \in N \cup I} f_j^*(\omega) = 0. & & &
\end{array}
\tag{4.7}
$$

System (4.7) is simply the set of all KKT-conditions corresponding to (4.3), (4.4), (4.5), (4.6). Note that in (4.7) information on how p_{ik}^{*N}, $k \in K^N$, $i \in I$, and p_{ik}^{*R}, $k \in K^R$, $i \in I$, relate to the price of electricity p_i^e in supply node $i \in I$ is suppressed. This relation is typically imposed by the regulator through a pricing scheme. Examples of such pricing schemes will be given in the remainder of this chapter when we introduce a renewable obligation and feed-in tariffs.

An equilibrium to the entire two-stage problem, including the first stage, can be found by solving another MCP. In order to do so, we first present the KKT-optimality conditions of the first stage for x^* and ζ^*. The KKT-optimality conditions for non-renewable technology $k \in K^N$ with optimization problem

(4.3), are given by:

$$0 \leq -E_{\omega}\left[\beta_{ik}^{*N}(\omega)\right] + \kappa_{ik}^{N} \qquad \perp \quad x_{ik}^{*N} \geq 0 \qquad \forall i \in I. \tag{4.8}$$

This condition follows from the first order conditions of (4.1); a detailed derivation can be found in Gürkan et al. (2013). For each $\omega \in \Omega$, $\beta_{ik}^{*N}(\omega)$ is the optimal scarcity rent $\beta_{ik}^{*N}(\omega)$ in (4.7), given $x = x^*$. x_{ik}^{*N} is the optimal investment quantity in non-renewable technology $k \in K^N$ in supply node $i \in I$. In particular, it can only be positive when the expected scarcity rents cover the unit investment cost.

The KKT-optimality conditions for renewable technology $k \in K^R$ with optimization problem (4.4), are given by:

$$\begin{aligned} 0 \leq -E_{\omega}\left[\frac{\partial F_{ik}(x_{ik}^{*R},\omega)}{\partial x_{ik}^{R}}\beta_{ik}^{*R}(\omega)\right] + \kappa_{ik}^{R'}(x_{ik}^{*R}) + \zeta_k^{*R} \qquad &\perp \quad x_{ik}^{*R} \geq 0 \qquad \forall i \in I \\ 0 \leq M_k^R - \sum_{i \in I} x_{ik}^{*R} \qquad &\perp \quad \zeta_k^{*R} \geq 0. \end{aligned} \tag{4.9}$$

The conditions in (4.9) follow from the first order conditions of (4.2). Again, for each $\omega \in \Omega$, $\beta_{ik}^{*R}(\omega)$ is the optimal $\beta_{ik}^{*R}(\omega)$ in (4.7) for $x = x^*$. x_{ik}^{*R} is the optimal investment quantity in renewable technology $k \in K^R$ in supply node $i \in I$. The interpretation of the term $(\partial F_{ik}(x_{ik}^{*R},\omega)/\partial x_{ik}^{R})\beta_{ik}^{*R}(\omega)$ is the following. $\beta_{ik}^{*R}(\omega)$ is the scarcity rent of available capacity in renewable technology $k \in K^R$ in supply node $i \in I$ in realization $\omega \in \Omega$ corresponding to a unit change in production. $\partial F_{ik}(x_{ik}^{*R},\omega)/\partial x_{ik}^{R}$ is the change in production corresponding to a unit change in investment. $(\partial F_{ik}(x_{ik}^{*R},\omega)/\partial x_{ik}^{R})\beta_{ik}^{*R}(\omega)$ is thus the scarcity rent corresponding to a unit change in investment at $x = x^*$. The first condition of (4.9) thus implies that, in order to have a positive investment in renewable technology $k \in K^R$ in supply node $i \in I$, the expected scarcity rent corresponding to a unit investment should cover the sum of the marginal investment cost and the scarcity rent of the ceiling, ζ_k^{*R}.

The equilibrium to the two-stage game can now be found by solving the MCP that finds x^* and ζ^{*R} that satisfy the first stage conditions (4.8) for all $k \in K^N$ and (4.9) for all $k \in K^R$, and that finds, for given x^* and for each possible realization $\omega \in \Omega$ a solution to the second stage optimality conditions in (4.7). We next introduce the modifications that need to be done when including a renewable obligation or a feed-in tariff system.

4.3 Introducing a Renewable Obligation and Certificate System

In order to meet long term quota, a regulator can impose a renewable obligation on the electricity market. An obligation can be imposed either on producers or on consumers. Since the former is more common, we assume the obligation is imposed on the producers. Compliance to the obligation target is typically shown in the form of certificates. For a unit production with a renewable resource, a producer receives one certificate. These certificates can be traded on a secondary market and hence have a certain value, the certificate price. At the end of each period, typically a year, firms need to show compliance to the target by having sufficient certificates.

Since the obligation holds for a certain period, we add an obligation condition to the first stage. We impose that, in expectation, a fraction ϕ of the total expected production should come from renewable resources, that is,

$$0 \leq E_\omega \left[(1-\phi) \sum_{i \in I} \sum_{k \in K^R} y_{ik}^R(\omega) - \phi \sum_{i \in I} \sum_{k \in K^N} y_{ik}^N(\omega) \right] \quad \perp \quad \nu \geq 0. \quad (4.10)$$

The obligation is perpendicular to the variable ν, which can be seen as the value of a unit production with a renewable resource. Since each unit production is rewarded with a certificate, ν thus also represents the certificate price a renewable technology receives on top of the electricity price. We impose the following pricing scheme at the second stage. In all $i \in I$ we let

$$\begin{aligned} p_{ik}^N(\omega) &= p_i^e(\omega) \qquad && \forall k \in K^N \\ p_{ik}^R(\omega) &= p_i^e(\omega) + \nu \qquad && \forall k \in K^R. \end{aligned} \quad (4.11)$$

Note that, contrary to the pricing schemes presented in Chapter 3, we do not express p^N and p^R in terms of the consumer price. Instead, we express all prices in terms of the electricity price p^e as this allows for a better comparison with the model that will be discussed in the next section.

We replace p^N and p^R in the first and second stage problems defined in the previous section. For given x and ν, the MCP that solves the second stage to optimality is now to find for any $\omega \in \Omega$, $y^*(\omega), \delta^*(\omega)$, $p^*(\omega), \beta^*(\omega), \lambda^{*+}(\omega)$, $\lambda^{*-}(\omega), \rho^*(\omega)$, and $f^*(\omega)$ that satisfy the following set of KKT-conditions:

$$0 \leq \beta_{ik}^{*N}(\omega) - p_i^{*e}(\omega) + c_{ik}^N \quad \perp \quad y_{ik}^{*N}(\omega) \geq 0 \quad \forall i \in I, k \in K^N \quad (4.12a)$$

$$0 \leq \beta_{ik}^{*R}(\omega) - p_i^{*e}(\omega) - \nu + c_{ik}^R \quad \perp \quad y_{ik}^{*R}(\omega) \geq 0 \quad \forall i \in I, k \in K^R \quad (4.12b)$$

$$0 \le x_{ik}^N - y_{ik}^{*N}(\omega) \quad \perp \quad \beta_{ik}^{*N}(\omega) \ge 0 \quad \forall i \in I, k \in K^N \tag{4.12c}$$

$$0 \le F_{ik}(x_{ik}^R, \omega) - y_{ik}^{*R}(\omega) \quad \perp \quad \beta_{ik}^{*R}(\omega) \ge 0 \quad \forall i \in I, k \in K^R \tag{4.12d}$$

$$0 \le VOLL - p_j^{*e}(\omega) \quad \perp \quad \delta_j^*(\omega) \ge 0 \quad \forall j \in N \cup I \tag{4.12e}$$

$$\begin{aligned} 0 \le \sum_{k \in K^N} y_{jk}^{*N}(\omega) + \sum_{k \in K^R} y_{jk}^{*R}(\omega) + \\ \delta_j^*(\omega) + f_j^*(\omega) - d_j(\omega) \quad \perp \quad p_j^{*e}(\omega) \ge 0 \quad \forall j \in N \cup I \end{aligned} \tag{4.12f}$$

$$0 \le h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j^*(\omega) \quad \perp \quad \lambda_l^{*+}(\omega) \ge 0 \quad \forall l \in L \tag{4.12g}$$

$$0 \le h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j^*(\omega) \quad \perp \quad \lambda_l^{*-}(\omega) \ge 0 \quad \forall l \in L \tag{4.12h}$$

$$\begin{aligned} \sum_{l \in L} PTDF_{l,j}(\lambda_l^{*+}(\omega) - \lambda_l^{*-}(\omega)) + \\ p_j^{*c}(\omega) - \rho^*(\omega) = 0 \quad \forall j \in N \cup I \end{aligned} \tag{4.12i}$$

$$\sum_{j \in N \cup I} f_j^*(\omega) = 0. \tag{4.12j}$$

This set of KKT conditions is equivalent to the following optimization problem, the so called Optimal Power Flow (OPF) problem:

$$\begin{aligned} & Z^{OBL}(x, \nu, \omega) := \\ & \min_{y(\omega), \delta(\omega), f(\omega)} \sum_{i \in I} \sum_{k \in K^N} c_{ik}^N y_{ik}^N(\omega) + \sum_{i \in I} \sum_{k \in K^R} (c_{ik}^R - \nu) y_{ik}^R(\omega) + VOLL \sum_{j \in N \cup I} \delta_j(\omega) \\ & \text{s.t.} \quad \begin{array}{lll} y_{ik}^N(\omega) \le x_{ik}^N & (\beta_{ik}^N(\omega)) & \forall i \in I, k \in K^N \\ y_{ik}^R(\omega) \le F_{ik}(x_{ik}^R, \omega) & (\beta_{ik}^R(\omega)) & \forall i \in I, k \in K^R \\ \sum_{k \in K^N} y_{jk}^N(\omega) + \sum_{k \in K^R} y_{jk}^R(\omega) + & & \\ \delta_j(\omega) + f_j(\omega) \ge d_j(\omega) & (p_j^e(\omega)) & \forall j \in N \cup I \\ \sum_{j \in N \cup I} f_j(\omega) = 0 & (\rho(\omega)) & \\ \sum_{j \in N \cup I} PTDF_{l,j} f_j(\omega) \le h_l & (\lambda_l^+(\omega)) & \forall l \in L \\ \sum_{j \in N \cup I} PTDF_{l,j} f_j(\omega) \ge -h_l & (\lambda_l^-(\omega)) & \forall l \in L \\ y_{ik}^N(\omega) \ge 0 & & \forall i \in I, k \in K^N \\ y_{ik}^R(\omega) \ge 0 & & \forall i \in I, k \in K^R \\ \delta_j(\omega) \ge 0 & & \forall j \in N \cup I. \end{array} \end{aligned} \tag{4.13}$$

The first stage problem consists of (4.1) for all $k \in K^N$ and (4.2) for all $k \in K^R$ with p^N and p^R replaced according to the pricing scheme (4.11), and (4.10).

An equilibrium to the two stage problem of the decentralized electricity market can now be found by determining $x^*, \nu^*, \zeta^*, y^*(\omega), \delta^*(\omega), p^*(\omega), \beta^*(\omega), \lambda^{*+}(\omega), \lambda^{*-}(\omega), \rho^*(\omega), f^*(\omega), \omega \in \Omega$, satisfying

$$0 \leq -E_\omega \left[\beta_{ik}^{*N}(\omega)\right] + \kappa_{ik}^N \quad \perp \quad x_{ik}^{*N} \geq 0 \quad \forall i \in I, k \in K^N \tag{4.14a}$$

$$0 \leq -E_\omega \left[\frac{\partial F_{ik}(x_{ik}^{*R}, \omega)}{\partial x_{ik}^R} \beta_{ik}^{*R}(\omega)\right] + \kappa_{ik}^{R'}(x_{ik}^{*R}) + \zeta_k^{*R} \quad \perp \quad x_{ik}^{*R} \geq 0 \quad \forall i \in I, k \in K^R \tag{4.14b}$$

$$0 \leq M_k^R - \sum_{i \in I} x_{ik}^{*R} \quad \perp \quad \zeta_k^{*R} \geq 0 \quad \forall k \in K^R \tag{4.14c}$$

$$0 \leq E_\omega \left[(1-\phi) \sum_{i \in I} \sum_{k \in K^R} y_{ik}^{*R}(\omega) - \phi \sum_{i \in I} \sum_{k \in K^N} y_{ik}^{*N}(\omega)\right] \quad \perp \quad \nu^* \geq 0 \tag{4.14d}$$

and (4.12) at $x = x^*$ and $\nu = \nu^*$ in each realization $\omega \in \Omega$.

4.4 Introducing a Feed-in Tariff System

Feed-in tariffs are per unit subsidies paid by the regulator to firms for producing with certain renewable technologies. The (additional) financial support is given to encourage development and investment in renewable technologies, eventually leading to cleaner generation mixtures. The feed-in tariff (FIT) is a price based instrument; that is, (part of the) prices paid to renewable electricity producers are fixed by the regulator. It is typically a fixed amount of money paid for a unit production with a renewable resource either instead of the price of electricity or on top of it. To avoid confusion, in this chapter we use the term feed-in tariff for the entire amount of money that is paid per unit production with a renewable resource. Feed-in tariffs are paid either from taxes or from mark-ups paid by consumers (referred to as ratepayers). Since we assume inelastic exogenous demand in this chapter, it is also natural to assume that feed-in tariffs are paid from taxes. A feed-in tariff can be price independent or price dependent. Price dependent policies are policies where the given tariff varies with the price of electricity; usually the higher the price, the lower the (necessary) financial support. In this chapter, we focus on one specific feed-in policy, namely the price independent fixed price policy.

The fixed price policy is the most popular feed-in policy applied across Europe, and is applied in for example Austria, Bulgaria, France, Germany, Ireland, Portugal, and Slovakia. For a complete overview, see Klein (2008). Under the fixed price policy, each unit production with a renewable resource is rewarded with a price that is fixed for a certain contract period, typically several years. This fixed price is paid to firms instead of the price of electricity, and is independent of the price of electricity itself. The fixed price can be different for different (renewable) technologies. Since consumers pay the market price of electricity for each unit consumed, the actual feed-in tariff paid by the regulator is the difference between the fixed price and the price of electricity.

We let $\bar{\tau}_k^R$ be the fixed feed-in tariff for technology $k \in K^R$. These fixed tariffs are typically set by the regulator, known to the firms, and thus can be seen as a parameter. However, for analytical purposes we allow it to be an endogenous variable as discussed below. We assume the technology specific tariff to be the same in all supply nodes $i \in I$. The pricing scheme imposed on the market is that in each node $i \in I$ we set in each $\omega \in \Omega$:

$$\begin{aligned} p_{ik}^N(\omega) &= p_i^e(\omega) && \forall k \in K^N \\ p_{ik}^R(\omega) &= \bar{\tau}_k^R && \forall k \in K^R. \end{aligned} \tag{4.15}$$

A fixed feed-in tariff in relation to the electricity price is depicted in Figure 4.1. We see that, while the per unit price paid to a renewable technology is fixed, the payment on top of the price of electricity and thus the cost incurred by the regulator is fluctuating.

We replace p^N and p^R in the first and second stage problems according to (4.15). For given x and any $\omega \in \Omega$, the MCP that solves the second stage to optimality is now to find $y^*(\omega), \delta^*(\omega), p^*(\omega), \beta^*(\omega), \lambda^{*+}(\omega), \lambda^{*-}(\omega), \rho^*(\omega)$, $f^*(\omega)$ that satisfy the following set of KKT-conditions:

$$0 \leq \beta_{ik}^{*N}(\omega) - p_i^{*e}(\omega) + c_{ik}^N \quad \perp \quad y_{ik}^{*N}(\omega) \geq 0 \quad \forall i \in I, k \in K^N \tag{4.16a}$$

$$0 \leq \beta_{ik}^{*R}(\omega) - \bar{\tau}_k^R + c_{ik}^R \quad \perp \quad y_{ik}^{*R}(\omega) \geq 0 \quad \forall i \in I, k \in K^R \tag{4.16b}$$

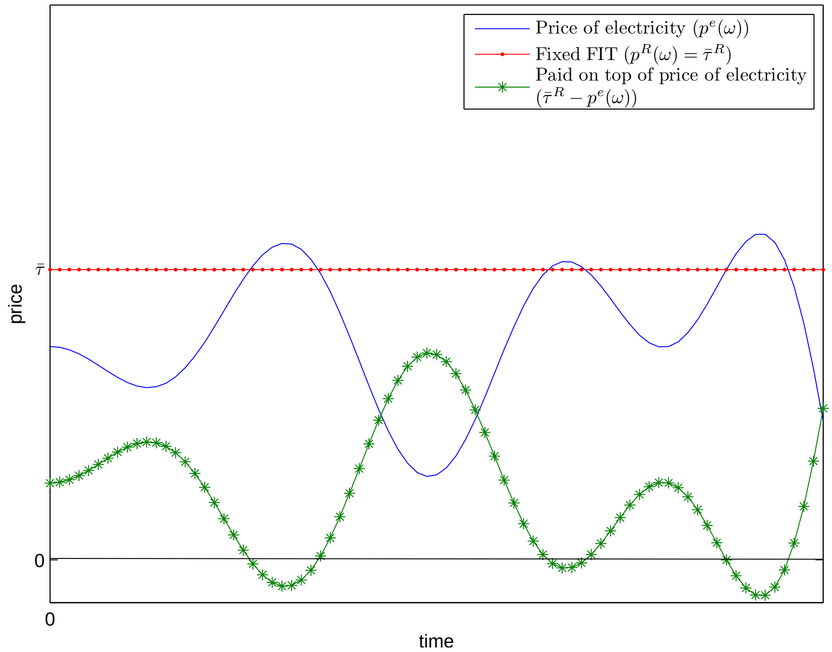
$$0 \leq x_{ik}^N - y_{ik}^{*N}(\omega) \quad \perp \quad \beta_{ik}^{*N}(\omega) \geq 0 \quad \forall i \in I, k \in K^N \tag{4.16c}$$

$$0 \leq F_{ik}(x_{ik}^R, \omega) - y_{ik}^{*R}(\omega) \quad \perp \quad \beta_{ik}^{*R}(\omega) \geq 0 \quad \forall i \in I, k \in K^R \tag{4.16d}$$

$$0 \leq VOLL - p_j^{*e}(\omega) \quad \perp \quad \delta_j^*(\omega) \geq 0 \quad \forall j \in N \cup I \tag{4.16e}$$

$$\begin{aligned} 0 \leq \sum_{k \in K^N} y_{jk}^{*N}(\omega) + \sum_{k \in K^R} y_{jk}^{*R}(\omega) + \\ \delta_j^*(\omega) + f_j^*(\omega) - d_j(\omega) \quad \perp \quad p_j^{*e}(\omega) \geq 0 \quad \forall j \in N \cup I \end{aligned} \tag{4.16f}$$

Figure 4.1: The fixed feed-in tariff policy.



$$0 \leq h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j^*(\omega) \quad \perp \quad \lambda_l^{*+}(\omega) \geq 0 \quad \forall l \in L \tag{4.16g}$$

$$0 \leq h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j^*(\omega) \quad \perp \quad \lambda_l^{*-}(\omega) \geq 0 \quad \forall l \in L \tag{4.16h}$$

$$\sum_{l \in L} PTDF_{l,j}(\lambda_l^{*+}(\omega) - \lambda_l^{*-}(\omega)) + p_j^{*c}(\omega) - \rho^*(\omega) = 0 \quad \forall j \in N \cup I \tag{4.16i}$$

$$\sum_{j \in N \cup I} f_j^*(\omega) = 0. \tag{4.16j}$$

In case of a fixed feed-in tariff, there is no equivalent OPF problem to (4.16). However, the problem can still be solved as an MCP. The first stage consists of (4.1) for all $k \in K^N$ and (4.2) for all $k \in K^R$ with p^N and p^R replaced according to the pricing scheme (4.15). An equilibrium to the two stage problem of the decentralized electricity market can now be found by determining $x^*, \zeta^*, y^*(\omega), \delta^*(\omega), p^*(\omega), \beta^*(\omega), \lambda^{*+}(\omega), \lambda^{*-}(\omega), \rho^*(\omega), f^*(\omega)$, $\omega \in \Omega$, satisfying

$$0 \leq -E_\omega \left[\beta_{ik}^{*N}(\omega)\right] + \kappa_{ik}^N \quad \perp \quad x_{ik}^{*N} \geq 0 \quad \forall i \in I, k \in K^N \tag{4.17a}$$

$$
\begin{aligned}
0 \leq -E_\omega \left[\frac{\partial F_{ik}(x_{ik}^{*R}, \omega)}{\partial x_{ik}^R} \beta_{ik}^{*R}(\omega) \right] + & \\
\kappa_{ik}^{R'}(x_{ik}^{*R}) + \zeta_k^{*R} \quad & \perp \quad x_{ik}^{*R} \geq 0 \quad \forall i \in I, k \in K^R \quad (4.17b) \\
0 \leq M_k^R - \sum_{i \in I} x_{ik}^{*R} \quad & \perp \quad \zeta_k^{*R} \geq 0 \quad \forall k \in K^R \quad (4.17c)
\end{aligned}
$$

and (4.16) at $x = x^*$ in each realization $\omega \in \Omega$.

In reality, regulators determine feed-in tariffs at the beginning of a period, often based on estimated levelised costs of renewable technologies, that is, the effective marginal costs for a unit production, including investment, fuel, operations and management, and other costs. Due to uncertainty in demand and uncertain output of renewable resources, the estimated levelised costs may differ from the actual costs involved with a unit production, resulting in either too much or insufficient financial support. In case too much support is given, this may result in windfall profits for renewable electricity producers and an excessive financial burden on the regulator and hence tax or ratepayers. When insufficient support is given, desired quota may not be met. Hence, a balance has to be found between giving too much and too little support.

Instead of an exogenous feed-in tariff based on levelised cost, in this chapter we aim to determine the feed-in tariff endogenously with the goal of meeting a renewable quota. In order to get a benchmark for the 'right' feed-in tariff, we impose a condition similar to the renewable obligation (4.10) at the first stage. For convenience, instead of imposing that a fraction ϕ of the total expected production should be produced with renewable technologies, we now impose that a fraction ϕ of the expected demand should be produced with renewables. In particular, we add the following constraint to the first-stage conditions (4.17):

$$
0 \leq E_\omega \left[\sum_{i \in I} \sum_{k \in K^R} y_{ik}^R(\omega) \right] - \phi E_\omega [d(\omega)] \quad \perp \quad \sigma \geq 0. \tag{4.18}
$$

Both (4.10) and (4.18) are similar, except when there is a lot of unsatisfied demand, in which case (4.18) requires more renewables than (4.10), or when there is a lot of excess production. In the latter case the electricity price drops to zero and (4.10) requires more renewables than (4.18). In practice and as observed in numerical experiments, unsatisfied demand is typically negligible and the electricity price dropping to zero is not very common. Hence, we can conclude that (4.18), while different, poses a good benchmark for verifying whether or not the original obligation on production (4.10) can be satisfied by the market.

The dual price σ obtained in (4.18) is not going to have the same interpretation as the price of certificates ν in (4.10), since the certificate price is paid on top of the electricity price while here the optimal dual price is going to represent the optimal unit payment that is paid instead of the electricity price. That is, we use σ to set the tariff levels. We introduce a parameter $\alpha_k \geq 0$ for each $k \in K^R$ with $\sum\limits_{k \in K^R} \alpha_k = 1$ and set $\bar{\tau}_k^R = \alpha_k \sigma$. These parameters are determined by the regulator and serve as a mean to discriminate between the different renewable technologies. This can be desirable in order to encourage investments in less established technologies or to limit investments in renewable technologies with certain negative side-effects.

4.5 Linear Versus Non-linear Convex Cost

In this section, we analyze the implications of both linear investment cost in renewable technologies and non-linear convex investment cost in renewable technologies. We focus on the fixed price policy and analyze whether the policy is able to guarantee that a renewable quota is met.

We consider an electricity market with only two available technologies, namely a non-renewable technology and a renewable technology. We assume there is a single node and hence do not take into account any network restrictions and in addition assume there is no limit on the investment quantity in both technologies. There is a random availability of renewable capacity at the second stage, denoted by the function $F^R(x^R, \omega)$, $\omega \in \Omega$, that is assumed to be of the following form: $F^R(x^R, \omega) = \theta(\omega) x^R$, $\omega \in \Omega$, with $\theta(\omega) \in (0, 1)$ representing the random availability according to a certain probability distribution. In addition, we make two assumptions on the cost. The second stage production cost, typically fuel cost, of the non-renewable producer is higher than the production cost of the renewable producer, that is, $c^N > c^R$. Furthermore, it is natural to assume that the expected effective marginal cost or levelised cost of the renewable producer are higher than the levelised cost of the non-renewable producer, that is,

$$\kappa^N + c^N < \frac{\kappa^{R'}(x^R)}{E_\omega[\theta(\omega)]} + c^R \quad \forall \quad x^R \geq 0. \tag{4.19}$$

The interpretation of the fraction involved in (4.19) is the following. Recall that $\kappa^R(x^R)$ is the investment cost function in x^R. $\kappa^{R'}(x^R)$ is its derivative with respect to x^R, and a unit investment in the renewable technology thus costs $\kappa^{R'}(x^R)$ in x^R. Of this unit, in expectation, only $E_\omega[\theta(\omega)] < 1$ is available. The expected investment cost of having one unit of production capacity available is thus $\frac{\kappa^{R'}(x^R)}{E_\omega[\theta(\omega)]}$.

We next show that with a renewable obligation and a certificate system, the set quota is always met at equilibrium. In addition, we give a lower bound for the optimal certificate price at an equilibrium. Before proving this, note that θ and β^{*R} or β^{*N} may be correlated and hence have a nonzero covariance. The following lemma shows that both covariances are nonpositive, which has a relevant implication later.

Lemma 4.1. *Let $\theta(\omega)$, $\omega \in \Omega$, be continuously distributed with values between 0 and 1. At a solution to the MCP consisting of (4.14) and (4.12) for given x and ν we have:*

(i) $COV_\omega(\theta(\omega), \beta^{*N}(\omega)) \leq 0$ *and consequently* $E_\omega[\theta(\omega)\beta^{*N}(\omega)] \leq E_\omega[\theta(\omega)]E_\omega[\beta^{*N}(\omega)]$.

(ii) $COV_\omega(\theta(\omega), \beta^{*R}(\omega)) \leq 0$ *and consequently* $E_\omega[\theta(\omega)\beta^{*R}(\omega)] \leq E_\omega[\theta(\omega)]E_\omega[\beta^{*R}(\omega)]$.

Proof: (i) We argue that θ and β^{*N} are negatively correlated and hence that their covariance is nonpositive. In each $\omega \in \Omega$, $\theta(\omega)$ represents the available capacity and hence affects $y^{*R}(\omega)$. Low values of $\theta(\omega)$ lead to low $y^{*R}(\omega)$. When there is sufficiently high demand, the non-renewable technology produces at full capacity and hence $\beta^{*N}(\omega)$ is determined by $p^{*e}(\omega) + c^N$. The electricity price is equal to the effective marginal cost of the non-renewable producer, unless there is unsatisfied demand; in that case, $p^{*e}(\omega) = VOLL$ and thus $\beta^{*N}(\omega)$ will be very high. On the other hand, high values of $\theta(\omega)$ lead to high $y^{*R}(\omega)$ and the non-renewable technology may not be able to produce at full capacity, leading to $\beta^{*N}(\omega) = 0$. Therefore, $\theta(\omega)$ and $\beta^{*N}(\omega)$ are negatively correlated. This implies $COV_\omega(\theta(\omega), \beta^{*N}(\omega)) \leq 0$. Then $E_\omega[\theta(\omega)\beta^{*N}(\omega)] = E_\omega[\theta(\omega)]E_\omega[\beta^{*N}(\omega)] + COV_\omega(\theta(\omega), \beta^{*N}(\omega)) \leq E_\omega[\theta(\omega)]E_\omega[\beta^{*N}(\omega)]$ follows immediately.

(ii) We argue that θ and β^{*R} are negatively correlated and hence that their covariance is nonpositive. In each $\omega \in \Omega$, when $x^{*R} > 0$ and hence $y^{*R}(\omega) > 0$, $\beta^{*R}(\omega) = p^{*e}(\omega) + \nu - c^R$. ν depends on the distribution of $\theta(\omega)$, but is the same in every $\omega \in \Omega$, c^R is a constant, and hence $\beta^{*R}(\omega)$ changes with $p^{*e}(\omega)$. In the proof of (i) we argued that $p^{*e}(\omega)$ is high when $\theta(\omega)$ is low.

We conclude that $\theta(\omega)$ and $\beta^{*R}(\omega)$ are negatively correlated. This implies $COV_\omega(\theta(\omega), \beta^{*R}(\omega)) \leq 0$. Then $E_\omega[\theta(\omega)\beta^{*R}(\omega)] = E_\omega[\theta(\omega)]E_\omega[\beta^{*R}(\omega)] + COV_\omega(\theta(\omega), \beta^{*R}(\omega)) \leq E_\omega[\theta(\omega)]E_\omega[\beta^{*R}(\omega)]$ follows immediately.□

Theorem 4.1. *Consider the MCP consisting of (4.14) at the first stage and (4.12) at the second stage for every $\omega \in \Omega$. The regulator thus imposes a renewable obligation (4.10) with $\phi \in [0, 1)$ on the market along with the pricing scheme (4.11). The following statements hold at an equilibrium:*

(i) $\phi = 0$ *implies* $x^{*R} = 0$.

(ii) $\phi > 0$ *implies that (4.10) is satisfied with equality.*

(iii) When $\phi > 0$, $\nu^* \geq \dfrac{\kappa^{R'}(x^{*R})}{E_\omega[\theta(\omega)]} + c^R - \kappa^N - c^N$.

Proof: (i) Suppose $x^{*R} > 0$. By (4.10), since $\phi = 0$, $\nu^* = 0$ or $y^{*R}(\omega) = 0$ in all $\omega \in \Omega$. If $\nu^* = 0$, then (4.12b) reduces to $0 \leq -p^{*e}(\omega) + \beta^{*R}(\omega) + c^R$ in all $\omega \in \Omega$. Together with $0 \leq -p^{*e}(\omega) + \beta^{*N}(\omega) + c^N$, $\omega \in \Omega$, and the assumption that $c^N > c^R$, this implies that $\beta^{*N}(\omega) < \beta^{*R}(\omega)$ in all $\omega \in \Omega$. Then we have in all $\omega \in \Omega$ that $y^{*R}(\omega) = \theta(\omega)x^{*R} > 0$ and hence $\beta^{*R}(\omega) = p^{*e}(\omega) - c^R$. This leads to the following:

$$E_\omega[\beta^{*R}(\omega)] + c^R = E_\omega[p^{*e}(\omega)] \leq E_\omega[\beta^{*N}(\omega) + c^N] \leq \kappa^N + c^N <$$

$$\frac{\kappa^{R'}(x^{*R})}{E_\omega[\theta(\omega)]} + c^R.$$

The first inequality follows from (4.12a), the second inequality follows from (4.8), and the last inequality follows from assumption (4.19). We conclude that

$$E_\omega[\beta^{*R}(\omega)] < \frac{\kappa^{R'}(x^{*R})}{E_\omega[\theta(\omega)]},$$

and using Lemma 4.1 that

$$E_\omega[\theta(\omega)\beta^{*R}(\omega)] \leq E_\omega[\theta(\omega)]E_\omega[\beta^{*R}(\omega)] < \kappa^{R'}(x^{*R}).$$

This implies that the left-hand side of (4.14b) holds with strict inequality, meaning that $x^{*R} = 0$ which is a contradiction with the assumption that $x^{*R} >$

0. If, on the other hand, $\nu^* > 0$ and $y^{*R}(\omega) = 0$ in all $\omega \in \Omega$, then $y^{*R}(\omega) < \theta(\omega)x^{*R}$ in all $\omega \in \Omega$. This implies that $\beta^{*R}(\omega) = 0$ in all $\omega \in \Omega$ and that the left-hand side of (4.14b) holds with strict inequality, leading to $x^{*R} = 0$ and again a contradiction.

(ii) Suppose that x^{*R} is such that (4.10) is not satisfied with equality. We have strict inequality and hence $\nu^* = 0$. By the same argument that was used in the proof of (i), this implies $x^{*R} = 0$, meaning that $y^{*R}(\omega) = 0$ in all $\omega \in \Omega$. Since $\phi > 0$, (4.10) is then either violated since there is no renewable production, or holds with equality when there is no production at all. The latter contradicts the assumption that (4.10) is not satisfied with equality. Hence, x^{*R} will be such that (4.10) is satisfied with equality.

(iii) Since $\phi < 1$, $x^{*N} > 0$ and hence by (4.14a) $E_\omega[\beta^{*N}(\omega)] = \kappa^N$. We know that $x^{*R} > 0$ and hence by (4.14b) that $E_\omega[\theta(\omega)\beta^{*R}(\omega)] = \kappa^{R'}(x^{*R})$. It holds that in all $\omega \in \Omega$, $y^{*R}(\omega) = \theta(\omega)x^{*R} > 0$, since if for some $\tilde{\omega} \in \Omega$ $y^{*R}(\tilde{\omega}) < \theta(\tilde{\omega})x^{*R}$, $\beta^{*R}(\tilde{\omega}) = 0$ and (4.12b) would be violated, that is, it would reduce to $-p^{*e}(\tilde{\omega}) - \nu^* + c^R < 0$, where the inequality holds by the fact that $p^{*e}(\omega) \geq c^N > c^R$. In all $\omega \in \Omega$, $y^{*R}(\omega) > 0$ and hence $\beta^{*R}(\omega) = p^{*e}(\omega) + \nu^* - c^R$. Furthermore, we know that by (4.12a) $p^{*e}(\omega) \leq \beta^{*N}(\omega) + c^N$. We get that

$$\kappa^{R'}(x^{*R}) = E_\omega[\theta(\omega)\beta^{*R}(\omega)] = E_\omega[\theta(\omega)(p^{*e}(\omega) + \nu^* - c^R)] \leq$$

$$E_\omega[\theta(\omega)(\beta^{*N}(\omega) + c^N + \nu^* - c^R)] \leq E_\omega[\theta(\omega)](\kappa^N + c^N + \nu^* - c^R).$$

The latter inequality follows from Lemma 4.1. Solving for ν^*, we obtain the given lower bound. □

Note that (iii) implies that ν^* is at least equal to the difference in levelised cost between the renewable and non-renewable technology. This is thus the fair price to pay to the renewable producer for investing in a more costly technology. With an optimal ν^* a new firm entering the market would be indifferent between investing in the non-renewable and the renewable technology. Due to the imposed obligation, firms would have to satisfy the obligation target and have no incentive to overinvest in the renewable technology as the certificate price would drop to zero.

We next consider the benchmark model for determining the feed-in tariffs, consisting of (4.16) for every $\omega \in \Omega$ at the second stage, and (4.17) and the renewable obligation (4.18) at the first stage. The renewable obligation is imposed solely for finding the equilibrium value of σ^*. Within the model, we set the fixed price $\bar{\tau}^R$ equal to σ^*, meaning that renewable technologies receive a fixed price of σ^* for each unit production. Here, σ is thus a variable for determining a certain fixed feed-in tariff, while $\bar{\tau}^R$ is the tariff, in this case equal to σ^*. Later, we remove the obligation constraint and analyze the effects of handing out a tariff that is different from the optimal σ^*, both in case of linear investment cost and non-linear convex investment cost in the renewable technology. Before doing so, we first state a useful lemma stating that the renewable technology is always producing at full (available) capacity.

Lemma 4.2. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$. At the equilibrium it holds that in each $\omega \in \Omega$ we have $y^{*R}(\omega) = \theta(\omega)x^{*R}$.*

Proof: We distinguish between two cases, namely $x^{*R} = 0$ and $x^{*R} > 0$. The first case is trivial since $y^{*R}(\omega) = 0$ should hold in each $\omega \in \Omega$. For $x^{*R} > 0$, suppose for some $\tilde{\omega} \in \Omega$ we have $y^{*R}(\tilde{\omega}) < \theta(\tilde{\omega})x^{*R}$, then $\beta^{*R}(\tilde{\omega}) = 0$ and (4.16b) reduces to $0 \leq -\bar{\tau}^R + c^R$. This is only feasible when $\bar{\tau}^R < c^R$, but in that case $\beta^{*R}(\omega) = 0$ in all $\omega \in \Omega$ and the left-hand side of (4.17b) would be a strict inequality, implicating $x^{*R} = 0$, which is a contradiction.□

Note that the model allows for (renewable) production to exceed demand in certain realizations. In particular, in realizations with significantly low demand and high output of renewable rescources, this could potentially occur. However, since in reality the percentage of renewable capacity is relatively low compared to non-renewable technologies and significant drops in electricity demand are not common, we do not explicitly exclude equilibria with excess production within the model.

Theorem 4.2. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$. Assume that in order to get a benchmark value for the feed-in tariffs, the regulator imposes a renewable obligation (4.18) with $\phi \in [0, 1)$ on the first stage, along with the pricing scheme for the fixed price feed-in policy (4.15). We set $\bar{\tau}^R = \sigma^*$ endogenously. The following statements hold at an equilibrium:*

(i) $\phi = 0$ *implies* $x^{*R} = 0$.

(ii) $\phi > 0$ *implies that (4.18) is satisfied with equality.*

(iii) *When* $\phi > 0$, $\sigma^* = \frac{\kappa^{R'}(x^{*R})}{E_\omega[\theta(\omega)]} + c^R$.

Proof: (i) Suppose $x^{*R} > 0$. By (4.18), since $\phi = 0$, $y^{*R}(\omega) = 0$ in all $\omega \in \Omega$ or $\sigma^* = 0$. The former is in contradiction with Lemma 4.3. If $\sigma^* = 0$, then (4.16b) reduces to $0 \leq \beta^{*R}(\omega) + c^R$ in all $\omega \in \Omega$. If $c^R > 0$, this implies that $0 = y^{*R}(\omega)$ in all $\omega \in \Omega$. Similarly, if $c^R = 0$, then in each $\omega \in \Omega$ either $\beta^{*R}(\omega) = 0$ or $y^{*R}(\omega) = 0$. However, if $\beta^{*R}(\omega) = 0$ in all $\omega \in \Omega$, then (4.17b) implies $x^{*R} = 0$, yielding a contradiction.

(ii) Suppose that x^{*R} is such that (4.18) is not satisfied with equality. This implies $\sigma^* = 0$, so (4.16b) again reduces to $0 \leq \beta^{*R}(\omega) + c^R$, and we again have in all $\omega \in \Omega$ that $\beta^{*R}(\omega) = 0$ or $y^{*R}(\omega) = 0$, implying $x^{*R} = 0$. Given that $\phi > 0$, in order for the problem to be feasible, $x^{*R} > 0$ should hold and hence (4.18) must be satisfied with equality.

(iii) We know that $x^{*R} > 0$ and hence by (4.17b) that $E_\omega[\theta(\omega)\beta^{*R}(\omega)] = \kappa^{R'}(x^{*R})$. Since $y^{*R}(\omega) > 0$ in all $\omega \in \Omega$, $\beta^{*R}(\omega) = \sigma^* - c^R$ in all $\omega \in \Omega$. We obtain

$$\kappa^{R'}(x^{*R}) = E_\omega[\theta(\omega)\beta^{*R}(\omega)] = E_\omega[\theta(\omega)(\sigma^* - c^R)] = E_\omega[\theta(\omega)](\sigma^* - c^R).$$

□

By (iii) in Theorem 4.2 we conclude that, at the equilibrium, σ^* is equal to the levelised cost of the renewable technology in $x^R = x^{*R}$. Combining Lemma 4.2 and Theorem 4.2 (ii), we obtain the following corollary that will be relevant later.

Corollary 4.1. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$. Assume that in order to get a benchmark value for the feed-in tariffs, the regulator imposes a renewable obligation (4.18) with $\phi \in [0, 1)$ on the first stage, along with the pricing scheme for the fixed price feed-in policy (4.15). Then at the equilibrium $x^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta(\omega)]}$.*

Proof: This follows directly from (ii) in Theorem 4.2 and Lemma 4.2. □

Next, we are going to consider linear investment cost functions for renewable technologies and remove the explicit obligation (4.18) that was used to get a benchmark value for the feed-in tariff from the set of optimality conditions. Instead, the feed-in tariffs are set exogenously based on the results in Theorem 4.2.

Theorem 4.3. *Consider the same model as in Theorem 4.2, but without the renewable obligation (4.18) at the first stage and consider the feed-in tariff $\bar{\tau}^R$ to be given exogenously. We assume a linear investment cost function, that is, $\kappa^R(x^R) = \kappa^R x^R$, for* *some constant $\kappa^R > 0$. Let σ^* be the optimal σ derived in Theorem 4.2. The following statements hold at an equilibrium:*

(i) For $\bar{\tau}^R < \sigma^*$, $x^{*R} = 0$.

(ii) For $\bar{\tau}^R > \sigma^*$, the problem is infeasible.

(iii) For $\bar{\tau}^R = \sigma^*$, any $x^{*R} \geq 0$ is feasible.

Proof: (i) If $\bar{\tau}^R < c^R$, then $\beta^{*R}(\omega) = 0$ in every $\omega \in \Omega$ and the left-hand side (4.17b) holds with strict inequality, yielding $x^{*R} = 0$. For $c^R \leq \bar{\tau}^R < \sigma^* = \frac{\kappa^R}{E_\omega[\theta(\omega)]} + c^R$, suppose that $x^{*R} > 0$. By Lemma 4.2 it follows that in every $\omega \in \Omega$ we have $y^{*R}(\omega) = \theta(\omega)x^{*R}$. Then (4.16b) implies $\beta^{*R}(\omega) = \bar{\tau}^R - c^R$ in all $\omega \in \Omega$. We obtain

$$E_\omega[\theta(\omega)\beta^{*R}(\omega)] = E_\omega[\theta(\omega)(\bar{\tau}^R - c^R)] < E_\omega[\theta(\omega)(\sigma^* - c^R)] =$$

$$E_\omega[\theta(\omega)(\frac{\kappa^R}{E_\omega[\theta(\omega)]})] = \kappa^R.$$

This means that the left-hand side of (4.17b) holds with strict inequality and hence that $x^{*R} = 0$. This contradicts our assumption.

(ii) $\bar{\tau}^R > \sigma^* = \frac{\kappa^R}{E_\omega[\theta(\omega)]} + c^R$. By (4.16b) we have that in each $\omega \in \Omega$ $\beta^{*R}(\omega) \geq \bar{\tau}^R - c^R > 0$. Equality is implied, since $\beta^{*R}(\omega) > 0$ means $y^{*R}(\omega) = \theta(\omega)x^{*R} > 0$ in all $\omega \in \Omega$. We get that

$$E_\omega[\theta(\omega)\beta^{*R}(\omega)] = E_\omega[\theta(\omega)(\bar{\tau}^R - c^R)] > E_\omega[\theta(\omega)(\sigma^* - c^R)] = \kappa^R.$$

This means that the left-hand side of (4.17b) is violated; the problem is infeasible.

(iii) Distinguish between $x^{*R} > 0$ and $x^{*R} = 0$. If $x^{*R} > 0$, then by Lemma 4.2 $y^{*R}(\omega) = \theta(\omega)x^{*R} > 0$ in every $\omega \in \Omega$. This yields $\beta^{*R}(\omega) = \sigma^* - c^R = \frac{\kappa^R}{E_\omega[\theta(\omega)]}$ in every $\omega \in \Omega$ and hence $E_\omega[\theta(\omega)\beta^{*R}(\omega)] = \kappa^R$. If $x^{*R} = 0$, then in every $\omega \in \Omega$ we have $y^{*R}(\omega) = 0$ and $\beta^{*R}(\omega) \geq \frac{\kappa^R}{E_\omega[\theta(\omega)]}$, meaning that (4.17b) is satisfied. It can in fact be checked that (4.16) and (4.17) are satisfied as long as x^{*N} is feasible. □

In reality, case (i) means that insufficient subsidies are paid for the renewable producer to cover her long term cost. Case (ii) means that too much support is given and that infinite investments in the renewable technology would incur with a tariff this high. Case (iii) is the result of removing the restrictive renewable obligation. A new firm entering the market would be indifferent between investing in the non-renewable technology or the renewable technology. In other words, in case of linear investment cost in the renewable technology, when a renewable obligation is not explicitly imposed on the market, feed-in tariffs cannot guarantee that a certain target on renewable production is met.

As mentioned in the introduction, a similar result is shown in Requate and Unold (2003) for a taxation in a model with a single decision stage and with no uncertainty. That result implies that subsidies would have the same effect. We have now shown that in a market with investments taking place at the first stage, production taking place at the second stage, and where both demand and availability of renewable resources at the second stage are random, a similar result holds for a fixed feed-in tariff.

We next show that when a strictly convex investment cost function for the renewable technology is considered, the obligation target can be met without explicitly imposing it on the market. We can achieve this by handing out a feed-in tariff that is equal to the dual variable that we find using our benchmark model. In addition, we show that when deviating from the optimal feed-in tariff, the optimal investment in the renewable technology increases or decreases, but does not necessarily lead to infeasibility or zero investments as has been shown for the linear cost function in Theorem 4.3.

Theorem 4.4. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$. Assume that we have a strictly convex investment cost function for the renewable technology. We assume $\kappa^R(0) = 0$, $\kappa^{R'}(x) > 0$ $\forall x \geq 0$, and $\kappa^{R''}(x) > 0 \; \forall x \geq 0$. Assume that imposing (4.18) with a certain target $\phi \in (0,1)$ leads to an equilibrium investment quantity in the renewable technology equal to $x^{*R}(\phi) = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta(\omega)]}$ with a corresponding dual price $\sigma^*(\phi) = \frac{\kappa^{R'}(x^{*R}(\phi))}{E_\omega[\theta(\omega)]} + c^R$. Assume that a certain feed-in tariff $\bar{\tau}^R$ leads to a certain investment quantity equal to x^{*R}. The following statements hold:*

(i) If $\bar{\tau}^R < \sigma^*(\phi)$, then $x^{*R} < x^{*R}(\phi)$ and the left-hand side of (4.18) is violated.

(ii) If $\bar{\tau}^R > \sigma^*(\phi)$, then $x^{*R} > x^{*R}(\phi)$ and the left-hand side of (4.18) holds with strict inequality.

(iii) If $\bar{\tau}^R = \sigma^*(\phi)$, then $x^{*R} = x^{*R}(\phi)$ and left-hand side of (4.18) holds with

equality.

Proof: (i) We show that $x^{*R}(\phi)$ is not an equilibrium, and that at the equilibrium x^{*R} must be smaller. Suppose we have an equilibrium investment of $x^{*R} = x^{*R}(\phi)$. In each realization $\omega \in \Omega$, by Lemma 4.2 we have $y^{*R}(\omega) = \theta(\omega)x^{*R}(\phi) > 0$ and hence $\beta^{*R}(\omega) = \bar{\tau}^R - c^R$. We obtain

$$E_\omega[\theta(\omega)\beta^{*R}(\omega)] = E_\omega[\theta(\omega)(\bar{\tau}^R - c^R)] < E_\omega[\theta(\omega)(\frac{\kappa^{R'}(x^{*R}(\phi))}{E_\omega[\theta(\omega)]})] =$$

$$\kappa^{R'}(x^{*R}(\phi)).$$

This means that the left-hand side of (4.17b) holds with strict inequality and hence that we are not at an equilibrium. It is however possible to find an $x^{*R} \in [0, x^{*R}(\phi))$ for which $y^{*R}(\omega) = \theta(\omega)x^{*R}$ and $\beta^{*R}(\omega) = \bar{\tau}^R - c^R$ in each $\omega \in \Omega$ such that

$$E_\omega[\theta(\omega)\beta^{*R}(\omega)] = E_\omega[\theta(\omega)(\bar{\tau}^R - c^R)] = \kappa^{R'}(x^{*R}).$$

In particular, since $\kappa^{R'}(x)$ is an invertible function,

$$x^{*R} = (\kappa^{R'})^{-1}(E_\omega[\theta(\omega)(\bar{\tau}^R - c^R)]) < (\kappa^{R'})^{-1}(E_\omega[\theta(\omega)(\sigma^*(\phi) - c^R)]) =$$

$$x^{*R}(\phi).$$

The inequality follows from strict convexity of the investment cost function $\kappa^R(x)$, implying that its first derivative is a strictly increasing function. Using Lemma 4.2 and Corollary 4.1 we obtain

$$E_\omega[y^{*R}(\omega)] = E_\omega[\theta(\omega)]x^{*R} < E_\omega[\theta(\omega)]x^{*R}(\phi) = \phi E_\omega[d(\omega)].$$

(ii) We show that $x^{*R}(\phi)$ is not an equilibrium, and that at the equilibrium x^{*R} must be larger. Suppose we have an equilibrium investment of $x^{*R} = x^{*R}(\phi)$. In each realization $\omega \in \Omega$, $y^{*R}(\omega) = \theta(\omega)x^{*R}(\phi)$ by Lemma 4.2 and hence $\beta^{*R}(\omega) = \bar{\tau}^R - c^R$. We obtain

$$E_\omega[\theta(\omega)\beta^{*R}(\omega)] = E_\omega[\theta(\omega)(\bar{\tau}^R - c^R)] > E_\omega[\theta(\omega)(\sigma^*(\phi) - c^R)] =$$

$$\kappa^{R'}(x^{*R}(\phi)).$$

This means that the left-hand side of (4.17b) is violated and that the renewable producer can invest more, up to x^{*R} such that

$$E_\omega[\theta(\omega)\beta^{*R}(\omega)] = E_\omega[\theta(\omega)(\bar{\tau}^R - c^R)] = \kappa^{R'}(x^{*R}).$$

Similar to the proof of (i), since $\kappa^{R'}(x)$ is an invertible and strictly increasing function,

$$x^{*R} = (\kappa^{R'})^{-1}(E_\omega[\theta(\omega)(\bar{\tau}^R - c^R)]) > x^{*R}(\phi).$$

Using Lemma 4.2 and Corollary 4.1 we obtain

$$E_\omega[y^{*R}(\omega)] = E_\omega[\theta(\omega)]x^{*R} > E_\omega[\theta(\omega)]x^{*R}(\phi) = \phi E_\omega[d(\omega)].$$

(iii) From (i) and (ii) it follows that when we have an investment quantity $x^R \neq x^{*R}(\phi)$, the expected scarcity rents $E_\omega[\theta(\omega)\beta^{*R}(\omega)]$ are either more or less than the marginal investment cost in x^R. With $\bar{\tau}^R = \sigma^*(\phi)$, obviously

$$E_\omega[\theta(\omega)\beta^{*R}(\omega)] = \kappa^{R'}(x^{*R}(\phi)),$$

and hence $x^{*R} = x^{*R}(\phi)$ is the equilibrium solution coinciding with the equilibrium solution we obtained in the benchmark model in Theorem 4.2. □

4.5.1 Multiple Renewable Technologies

Next, we consider an electricity market with a single non-renewable technology and several renewable technologies, namely $|K^R| > 1$. We still assume there is a single node and hence do not take into account any network restrictions and assume there is no limit on the investment quantity in the technologies. Random availability of renewable capacities at the second stage are still of the form $F_k^R(x_k^R, \omega) = \theta_k(\omega)x_k^R$, $k \in K^R$, $\omega \in \Omega$, with $\theta_k(\omega) \in (0,1)$ following a certain probability distribution. We make the following assumption on the order of the levelised cost:

$$\kappa^N + c^N < \frac{\kappa_k^{R'}(x_k^R)}{E_\omega[\theta_k(\omega)]} + c_k^R \quad \forall \quad x_k^R \geq 0, k \in K^R. \tag{4.20}$$

We thus assume that all renewable technologies have higher levelised cost than the non-renewable technology. We next show some properties that have valuable implications. First we show that at the equilibrium all renewable technologies produce at full capacity in each realization. This is a result similar to Lemma 4.2 and is stated for the sake of completeness since it will be useful in the rest of this section.

Lemma 4.3. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$. At the equilibrium it holds that in each $\omega \in \Omega$ and for each $k \in K^R$ we have $y_k^{*R}(\omega) = \theta_k(\omega)x_k^{*R}$.*

Proof: Let $k \in K^R$ and apply the proof of Lemma 4.2.□

The following corollary is similar to Corollary 4.1 and has important implications later on.

Corollary 4.2. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$. Assume that in order to get a benchmark value for the feed-in tariffs, the regulator imposes a renewable obligation (4.18) with $\phi \in (0,1)$ on the first stage, along with the pricing scheme for the fixed price feed-in policy (4.15). Then at an equilibrium we have*

$$E_\omega\left[\sum_{k\in K^R} \theta_k(\omega)x_k^{*R}\right] = \phi E_\omega[d(\omega)]. \tag{4.21}$$

Proof: Since $\phi > 0$, at least one renewable technology will have a positive investment. For this to be possible, $\sigma^* > 0$ and (4.21) follows immediately from Lemma 4.3 and (4.18). □

Next, we analyze the equilibrium in case of linear investment cost. We assume $\kappa_k^R(x_k^R) = \kappa_k^R x_k^R$ for all $k \in K^R$, with some constants $\kappa_k^R > 0$, $k \in K^R$. Hence, marginal investment cost are constant and we can simply order the levelised cost for all technologies; we assume

$$\frac{\kappa_i^R}{E_\omega[\theta_i(\omega)]} + c_i^R < \frac{\kappa_j^R}{E_\omega[\theta_j(\omega)]} + c_j^R \quad \forall i < j. \tag{4.22}$$

We give each renewable technology $k \in K^R$ a subsidy of α_k times the optimal dual price σ^* determined by the renewable obligation constraint imposed at the first stage. This σ^* serves as a benchmark for determining the feed-in tariffs in a way that the renewable quota can be met. The α_ks are determined by the regulator, and we show for which choices of these parameters which technologies are in the optimal technology mixture.

Theorem 4.5. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$. Assume that in order to get a benchmark* *value for the feed-in tariffs, the regulator imposes a renewable obligation (4.18) with $\phi \in [0,1)$ on the first stage, along with the pricing scheme for the fixed price feed-in policy (4.15). We endogenously set $\bar{\tau}_k^R = \alpha_k \sigma^*$, $k \in K^R$, with α_k, $k \in K^R$, given and $\sum_{k=1}^{n} \alpha_k = 1$. Assume linear investment cost functions for all renewable technologies, that is $\kappa_k^R(x_k^R) = \kappa_k^R x_k^R$, $k \in K^R$. The following statements hold at an equilibrium:*

(i) *$\phi = 0$ implies $x_k^{*R} = 0 \; \forall k \in K^R$.*

(ii) *When $\phi > 0$ and $\alpha_1 \geq \alpha_k \; \forall \, k \in K^R$, then $x_1^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_1(\omega)]}$ and $x_k^{*R} = 0$ for all $k \neq 1$.*

(iii) *For every $h \in K^R$, when $\alpha_k = 0$ for all $k \neq 1, h$, there exists unique α_1^* and α_h^*, such that $x_1^{*R} > 0$ and $x_h^{*R} > 0$ can occur, namely*

$$\alpha_1^* = \frac{\frac{\kappa_1^R}{E_\omega[\theta_1(\omega)]} + c_1^R}{\frac{\kappa_1^R}{E_\omega[\theta_1(\omega)]} + c_1^R + \frac{\kappa_h^R}{E_\omega[\theta_h(\omega)]} + c_h^R} \tag{4.23}$$

and

$$\alpha_h^* = \frac{\frac{\kappa_h^R}{E_\omega[\theta_h(\omega)]} + c_h^R}{\frac{\kappa_1^R}{E_\omega[\theta_1(\omega)]} + c_1^R + \frac{\kappa_h^R}{E_\omega[\theta_h(\omega)]} + c_h^R}. \tag{4.24}$$

Proof: (i) This follows immediately from the proof of (i) in Theorem 4.2, using assumptions (4.20) and (4.22).

(ii) Suppose there exists an $h \in K^R$, $h \neq 1$, for which $x_h^{*R} > 0$. By Lemma 4.3, $y_h^{*R}(\omega) = \theta_h(\omega) x_h^{*R} > 0 \; \forall \, \omega \in \Omega$, implying $\beta_h^{*R}(\omega) = \alpha_h \sigma^* - c_h^R$, $\omega \in \Omega$. On the other hand, since $x_h^{*R} > 0$, by (4.17b) we must have

$$\kappa_h^R = E_\omega[\theta_h(\omega)\beta_h^{*R}(\omega)] = E_\omega[\theta_h(\omega)(\alpha_h \sigma^* - c_h^R)].$$

From this we get an expression for $\alpha_h \sigma^*$, and using $\alpha_1 \geq \alpha_h$ and assumption (4.22) we have

$$\alpha_1 \sigma^* \geq \alpha_h \sigma^* = \frac{\kappa_h^R}{E_\omega[\theta_h(\omega)]} + c_h^R > \frac{\kappa_1^R}{E_\omega[\theta_1(\omega)]} + c_1^R.$$

This implies

$$E_\omega[\theta_1(\omega)\beta_1^{*R}(\omega)] \geq E_\omega[\theta_1(\omega)(\alpha_1\sigma^* - c_1^R)] > \kappa_1^R.$$

Hence, the left-hand side of (4.17b) is violated for $k = 1$. x_1^{*R} follows from Corollary 4.2.

(iii) Since $\alpha_k = 0$ for all $k \neq 1, h$, $\bar{\tau}_k^R = 0$ and consequently $x_k^{*R} = 0$ for all $k \neq 1, h$. We are going to find α_1, α_h with $\alpha_1 + \alpha_h = 1$ such that the left-hand side of (4.17b) holds with equality for both $k = 1$ and $k = h$. The latter implies that

$$\alpha_1\sigma^* = \frac{\kappa_1^R}{E_\omega[\theta_1(\omega)]} + c_1^R \tag{4.25}$$

and

$$\alpha_h\sigma^* = \frac{\kappa_h^R}{E_\omega[\theta_h(\omega)]} + c_h^R. \tag{4.26}$$

Using the fact that $\alpha_1 + \alpha_h = 1$, solving for σ^* yields

$$\tilde{\sigma}^* = \frac{\kappa_1^R}{E_\omega[\theta_1(\omega)]} + c_1^R + \frac{\kappa_h^R}{E_\omega[\theta_h(\omega)]} + c_h^R. \tag{4.27}$$

Using (4.27), α_1^* and α_h^* immediately follow from (4.25) and (4.26), respectively. □

Corollary 4.3. *Consider the problem in Theorem 4.5. Let α_1^* be defined by (4.23), α_h^* by (4.24), and $\tilde{\sigma}^*$ by (4.27). Then the following statements hold at an equilibrium:*

(i) If $\alpha_1 = \alpha_1^*$ and $\alpha_h = \alpha_h^*$, then $\sigma^* = \tilde{\sigma}^*$.

(ii) If $\alpha_h > \alpha_h^*$ and $\alpha_1 + \alpha_h = 1$, then $\sigma^* < \tilde{\sigma}^*$, $x_1^{*R} = 0$, and $x_h^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_h(\omega)]}$.

(iii) If $\alpha_h < \alpha_h^*$ and $\alpha_1 + \alpha_h = 1$, then $\sigma^* < \tilde{\sigma}^*$, $x_1^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_1(\omega)]}$, and $x_h^{*R} = 0$.

Proof: (i) Suppose $\sigma^* < \tilde{\sigma}^*$. Then $\alpha_1^*\sigma^* < \frac{\kappa_1^R}{E_\omega[\theta_1(\omega)]} + c_1^R$ and $\alpha_h^*\sigma^* < \frac{\kappa_h^R}{E_\omega[\theta_h(\omega)]} + c_h^R$. By (4.17b) we have $x_k^{*R} = 0$ for all $k \in K^R$ and (4.18) will be violated. Similarly, if $\sigma^* > \tilde{\sigma}^*$, then the left-hand side of (4.17b) is violated for $k = 1, h$.

(ii) Since $\alpha_1 + \alpha_h = 1$, $\alpha_1 < \alpha_1^*$. In order to satisfy the quota, at least one of (4.25) and (4.26) must hold. Suppose $\sigma^* \geq \tilde{\sigma}^*$ such that (4.25) is still satisfied regardless of the lower α_1. Then $\alpha_h\sigma^* > \alpha_h^*\sigma^* \geq \alpha_h^*\tilde{\sigma}^* = \frac{\kappa_h^R}{E_\omega[\theta_h(\omega)]} + c_h^R$, and hence the left-hand side of (4.17b) is violated for $k = h$. Hence, $\sigma^* < \tilde{\sigma}^*$ such

that (4.26) holds while $\alpha_1\sigma^* < \alpha_1^*\sigma^* < \alpha_1^*\tilde{\sigma}^* = \frac{\kappa_1^R}{E_\omega[\theta_1(\omega)]} + c_1^R$. The latter implies $x_1^{*R} = 0$. x_h^{*R} follows from Corollary 4.2.

(iii) Since $\alpha_1 + \alpha_h = 1$, $\alpha_1 > \alpha_1^*$. In order to satisfy the quota, again at least one of (4.25) and (4.26) must hold. Suppose $\sigma^* \geq \tilde{\sigma}^*$ such that (4.26) is still satisfied regardless of the lower α_h. Then $\alpha_1\sigma^* > \alpha_1^*\sigma^* \geq \alpha_1^*\tilde{\sigma}^* = \frac{\kappa_1^R}{E_\omega[\theta_1(\omega)]} + c_1^R$, and hence the left-hand side of (4.17b) is violated for $k = 1$. Hence, $\sigma^* < \tilde{\sigma}^*$ such that (4.25) holds while $\alpha_h\sigma^* < \alpha_h^*\sigma^* < \alpha_h^*\tilde{\sigma}^* = \frac{\kappa_h^R}{E_\omega[\theta_h(\omega)]} + c_h^R$. The latter implies $x_h^{*R} = 0$. x_1^{*R} follows from Corollary 4.2. □

Corollary 4.4. *Consider the problem in Theorem 4.5 and Corollary 4.3. For any subset $H \subseteq K^R$, if we take*

$$\alpha_k = \frac{\frac{\kappa_k^R}{E_\omega[\theta_k(\omega)]} + c_k^R}{\sum_{h\in H}\left[\frac{\kappa_h^R}{E_\omega[\theta_h(\omega)]} + c_h^R\right]} \quad \forall k \in H \tag{4.28}$$

and

$$\alpha_k = 0 \quad \forall k \notin H, \tag{4.29}$$

then for all $k \in H$ we can have $x_k^{*R} > 0$, for all $k \notin H$ we have $x_k^{*R} = 0$, and $\sigma^* = \sum_{h\in H}\left[\frac{\kappa_h^R}{E_\omega[\theta_h(\omega)]} + c_h^R\right].$

Proof: This follows immediately from (*iii*) in Theorem 4.5 applied to more than two technologies.□

Corollary 4.5. *Consider the problem in Corollary 4.4. If the explicit obligation (4.18) is removed from the problem statement, choosing the αs based on (4.28) and (4.29) cannot guarantee that the obligation will be implicitly satisfied. That is, any solution with $x_k^{*R} \geq 0\ \forall k \in H$, $x_k^{*R} = 0\ \forall k \notin H$, and $x^{*N} \geq 0$ for which the second stage constraints (4.16) are satisfied, are equilibrium solutions.*

Proof: This follows immediately from (*iii*) in Theorem 4.3 applied to multiple renewable technologies.□

From (*iii*) in Theorem 4.5 and Corollary 4.4 for $|H| \geq 2$, it follows that in case of linear cost functions, there are multiple equilibria as all technologies have their levelised cost covered, while there is no criterion to choose one technology over the other. Not all of these equilibria will satisfy the obligation when it is removed as an explicit constraint. Note that in reality, network limitations and limitations on the maximum installed capacity may play a role in technology choices and exclude some of the multiple equilibria that are found using a mathematical framework. On the other hand, when investment costs are considered to be non-linear and convex, we do obtain a unique equilibrium, as we show next.

Since with non-linear convex investment costs, determining the parameters for which technologies may be in the optimal mixture is more involved compared to the linear investment cost case, we first consider a market with a single non-renewable and only two renewable technologies. Both renewable technologies receive a per unit subsidy being $\bar{\tau}_k^R = \alpha_k \sigma^*$, $k = 1, 2$, with $\alpha_1 + \alpha_2 = 1$. To simplify notation, we take $\alpha_1 = \alpha$, $\alpha_2 = 1 - \alpha$, $\alpha \in [0, 1]$. Then, for each $\alpha \in [0, 1]$ we can consider the optimal technology mixture. In particular, for low values of α, technology 1 will not get sufficient support and only technology 2 will have a positive investment. The corresponding optimal investment quantity can be found using Corollary 4.2. For high values of α, technology 2 will not get sufficient support and the optimal investment quantity in technology 1 can be found using Corollary 4.2. For all remaining values of α, both technologies have a positive investment. The next theorem finds the maximum value of α for which only technology 2 is investing and the minimum value of α for which only technology 1 is investing.

Theorem 4.6. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$ and let $K^R = \{1, 2\}$. Assume that in order to get a benchmark value for the feed-in tariffs, the regulator imposes a renewable obligation (4.18) with $\phi \in (0, 1)$ on the first stage, along with the pricing scheme for the fixed price feed-in policy (4.15). We endogenously set $\bar{\tau}_1^R = \alpha\sigma^*$ and $\bar{\tau}_2^R = (1-\alpha)\sigma^*$ with $\alpha \in [0, 1]$. Assume non-linear convex investment cost functions for both renewable technologies. The following statements hold at an equilibrium:*

(i) If $\alpha \le \alpha_1^* := \dfrac{\frac{\kappa_1^{R\prime}(0)}{E_\omega[\theta_1(\omega)]} + c_1^R}{\frac{\kappa_1^{R\prime}(0)}{E_\omega[\theta_1(\omega)]} + c_1^R + \frac{\kappa_2^{R\prime}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_2(\omega)]}\right)}{E_\omega[\theta_2(\omega)]} + c_2^R}$*, then* $x_1^{*R} = 0$ *and* $x_2^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_2(\omega)]}$.

(ii) If $\alpha \ge \alpha_2^* := \dfrac{\frac{\kappa_1^{R\prime}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_1(\omega)]}\right)}{E_\omega[\theta_1(\omega)]} + c_1^R}{\frac{\kappa_1^{R\prime}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_1(\omega)]}\right)}{E_\omega[\theta_1(\omega)]} + c_1^R + \frac{\kappa_2^{R\prime}(0)}{E_\omega[\theta_2(\omega)]} + c_2^R}$*, then* $x_1^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_1(\omega)]}$ *and* $x_2^{*R} = 0$.

(iii) If $\alpha_1^* < \alpha < \alpha_2^*$*, then* $x_1^{*R} > 0$ *and* $x_2^{*R} > 0$.

Proof: (i) Since $\phi > 0$, at least one of the renewable technologies will have a positive investment. Suppose $x_1^{*R} > 0$. By (4.17b) we must have

$$E_\omega[\theta_1(\omega)\beta_1^{*R}(\omega)] = \kappa_1^{R'}(x_1^{*R}) \tag{4.30}$$

and

$$E_\omega[\theta_2(\omega)\beta_2^{*R}(\omega)] \leq \kappa_2^{R'}(x_2^{*R}). \tag{4.31}$$

For technology 2, by (4.16b) we know $\beta_2^{*R}(\omega) \geq (1-\alpha)\sigma^* - c_2^R$, $\omega \in \Omega$. Using this and (4.31) we obtain

$$\kappa_2^{R'}(x_2^{*R}) \geq E_\omega[\theta_2(\omega)]((1-\alpha)\sigma^* - c_2^R).$$

This leads to an upperbound for σ^* as a function of α, namely

$$\sigma^* \leq \frac{\kappa_2^{R'}(x_2^{*R})}{(1-\alpha)E_\omega[\theta_2(\omega)]} + \frac{c_2^R}{(1-\alpha)}. \tag{4.32}$$

Furthermore, for technology 1 we have $y_1^{*R}(\omega) = \theta_1(\omega)x_1^{*R} > 0$ in each $\omega \in \Omega$ by Lemma 4.3, and hence $\beta_1^{*R}(\omega) = \alpha\sigma^* - c_1^R$, $\omega \in \Omega$. Using (4.32) and since $\alpha \leq \alpha_1^*$, we obtain

$$E_\omega[\theta_1(\omega)\beta_1^{*R}(\omega)] = E_\omega[\theta_1(\omega)](\alpha\sigma^* - c_1^R) \leq$$

$$E_\omega[\theta_1(\omega)]\left(\frac{\alpha\kappa_2^{R'}(x_2^{*R})}{(1-\alpha)E_\omega[\theta_2(\omega)]} + \frac{\alpha c_2^R}{(1-\alpha)} - c_1^R\right) \leq$$

$$E_\omega[\theta_1(\omega)]\left(\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\kappa_2^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_2(\omega)]}\right)}{E_\omega[\theta_2(\omega)]} + c_2^R\right) - c_1^R\right) \leq$$

$$E_\omega[\theta_1(\omega)]\left(\frac{\frac{\kappa_1^{R'}(0)}{E_\omega[\theta_1(\omega)]} + c_1^R}{\frac{\kappa_2^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_2(\omega)]}\right)}{E_\omega[\theta_2(\omega)]} + c_2^R}\left(\frac{\kappa_2^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_2(\omega)]}\right)}{E_\omega[\theta_2(\omega)]} + c_2^R\right) - c_1^R\right) =$$

$$\kappa_1^{R'}(0) < \kappa_1^{R'}(x_1^{*R}).$$

We get that (4.30) is violated. Since $x_1^{*R} = 0$, by Corollary 4.2 $x_2^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_2(\omega)]}$.

(ii) Again, since $\phi > 0$, at least one of the renewable technologies will have

a positive investment. Suppose $x_2^{*R} > 0$. By (4.17b) we must have

$$E_\omega[\theta_1(\omega)\beta_1^{*R}(\omega)] \leq \kappa_1^{R'}(x_1^{*R}) \tag{4.33}$$

and

$$E_\omega[\theta_2(\omega)\beta_2^{*R}(\omega)] = \kappa_2^{R'}(x_2^{*R}). \tag{4.34}$$

For technology 1, by (4.16b) we know $\beta_1^{*R}(\omega) \geq \alpha\sigma^* - c_1^R$, $\omega \in \Omega$. Using this and (4.33) we obtain

$$\kappa_1^{R'}(x_1^{*R}) \geq E_\omega[\theta_1(\omega)](\alpha\sigma^* - c_1^R).$$

This leads to an upperbound for σ^* as a function of α, namely

$$\sigma^* \leq \frac{\kappa_1^{R'}(x_1^{*R})}{\alpha E_\omega[\theta_1(\omega)]} + \frac{c_1^R}{\alpha}. \tag{4.35}$$

Furthermore, for technology 2 we have $y_2^{*R}(\omega) = \theta_2(\omega)x_2^{*R} > 0$ in each $\omega \in \Omega$ by Lemma 4.3, and hence $\beta_2^{*R}(\omega) = (1-\alpha)\sigma^* - c_2^R$, $\omega \in \Omega$. Using (4.35) and since $\alpha \geq \alpha_2^*$, we obtain

$$\begin{gathered}
E_\omega[\theta_2(\omega)\beta_2^{*R}(\omega)] = E_\omega[\theta_2(\omega)]((1-\alpha)\sigma^* - c_2^R) \leq \\
E_\omega[\theta_2(\omega)]\left(\frac{(1-\alpha)\kappa_1^{R'}(x_1^{*R})}{\alpha E_\omega[\theta_1(\omega)]} + \frac{(1-\alpha)c_1^R}{\alpha} - c_2^R\right) \leq \\
E_\omega[\theta_2(\omega)]\left((\frac{1}{\alpha} - 1)\left(\frac{\kappa_1^{R'}(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_1(\omega)]})}{E_\omega[\theta_1(\omega)]} + c_1^R\right) - c_2^R\right) \leq \\
E_\omega[\theta_2(\omega)]\left(\frac{\frac{\kappa_2^{R'}(0)}{E_\omega[\theta_2(\omega)]} + c_2^R}{\frac{\kappa_1^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_1(\omega)]}\right)}{E_\omega[\theta_1(\omega)]} + c_1^R}\left(\frac{\kappa_1^{R'}(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_1(\omega)]})}{E_\omega[\theta_1(\omega)]} + c_1^R\right) - c_2^R\right) = \\
\kappa_2^{R'}(0) < \kappa_2^{R'}(x_2^{*R}).
\end{gathered}$$

We get that (4.34) is violated. Since $x_2^{*R} = 0$, by Corollary 4.2 $x_1^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_1(\omega)]}$.

(iii) Suppose $x_1^{*R} > 0$ and $x_2^{*R} = 0$. Then $x_1^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_1(\omega)]}$ and we must have

$$E_\omega[\theta_1(\omega)\beta_1^{*R}(\omega)] = \kappa_1^{R'}(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_1(\omega)]}) \tag{4.36}$$

and

$$E_\omega[\theta_2(\omega)\beta_2^{*R}(\omega)] \leq \kappa_2^{R'}(0). \tag{4.37}$$

Using that $\beta_1^{*R}(\omega) = \alpha\sigma^* - c_1^R$ for all $\omega \in \Omega$, we can use (4.36) to obtain an expression for σ^*. Using this and the fact that $\beta_2^{*R}(\omega) \geq (1-\alpha)\sigma^* - c_2^R$, $\omega \in \Omega$, and $\alpha < \alpha_2^*$ yields

$$E_\omega[\theta_2(\omega)\beta_2^{*R}(\omega)] \geq E_\omega[\theta_2(\omega)]((1-\alpha)\sigma^* - c_2^R) =$$

$$E_\omega[\theta_2(\omega)]\left(\frac{(1-\alpha)\kappa_1^{R'}(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_1(\omega)]})}{\alpha E_\omega[\theta_1(\omega)]} + \frac{(1-\alpha)c_1^R}{\alpha} - c_2^R\right) >$$

$$E_\omega[\theta_2(\omega)]\left(\frac{\frac{\kappa_2^{R'}(0)}{E_\omega[\theta_2(\omega)]} + c_2^R}{\frac{\kappa_1^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_1(\omega)]}\right)}{E_\omega[\theta_1(\omega)]} + c_1^R}\left(\frac{\kappa_1^{R'}(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_1(\omega)]})}{E_\omega[\theta_1(\omega)]} + c_1^R\right) - c_2^R\right) = \kappa_2^{R'}(0).$$

Hence, (4.37) is violated. In a similar way, $x_1^{*R} = 0$ leads to a contradiction. □

Note that due to assumption (4.20) we have that $\frac{\kappa_k^{R'}(0)}{E_\omega[\theta_k(\omega)]} + c_k^R > \kappa^N + c^N \geq 0$ for $k = 1,2$. This means that the case where $\kappa_1^{R'}(0) = 0$ and $c_1^R = 0$, which would imply that the first technology is in the optimal mixture for all $\alpha > 0$, cannot occur. In addition, the case where $\kappa_2^{R'}(0) = 0$ and $c_2^R = 0$, which would imply that the second technology is in the optimal mixture for all $\alpha < 1$, cannot occur either.

We next consider a market with a single non-renewable and three renewable technologies. The renewable technologies receive a per unit subsidy being $\bar{\tau}_k^R = \alpha_k\sigma^*$, $k = 1,2,3$, with $\alpha_1 + \alpha_2 + \alpha_3 = 1$ and $\alpha_k \geq 0$, $k = 1,2,3$. We are going to consider vectors $\alpha = [\alpha_1 \quad \alpha_2 \quad \alpha_3]^\top$ and their corresponding optimal technology mixture. In particular, we are going to find such vectors for which one technology has a positive investment while the other two have zero investment. Using these vectors, we can characterize the equilibrium in case of quadratic cost functions as we discuss later.

First, consider vectors $\alpha = [\alpha_1 \quad \alpha_2 \quad \alpha_3]^\top$ with one of the elements equal to zero. Obviously, the renewable technology corresponding to the zero element will not invest. Theorem 4.6 can then be applied to the remaining two technologies. For example, we could take $\alpha_3 = 0$ and apply Theorem 4.6 to technologies 1 and 2. We then obtain two vectors, namely $\alpha^{2,3} = [\alpha_1^* \quad 1-\alpha_1^* \quad 0]^\top$ and $\alpha^{1,3} = [\alpha_2^* \quad 1-\alpha_2^* \quad 0]^\top$ with α_1^* and α_2^* as defined in Theorem 4.6. The meaning of the vector $\alpha^{2,3}$ is then the following: when $\alpha_3 = 0$, α_1^* is the largest α_1 for which $x_1^{*R} = 0$ and technology 2 is the sole investor. Slightly increasing α_1 and decreasing α_2 results in positive investment in both technologies 1 and 2. The meaning of the vector $\alpha^{1,3}$ is the following: when $\alpha_3 = 0$, α_2^* is the smallest α_1 for which $x_2^{*R} = 0$ and technology 1 is the sole investor. Slightly decreasing α_1 and increasing α_2 results in positive investment in both technologies 1 and 2. The first index in the superscript denotes the technology that has a positive investment, while the second index indicates the technology for which the subsidy is zero. Formally, we define all six such vectors as follows:

The vector $\alpha^{i,h} \in \mathbb{R}^3$ is given by

$$\alpha_i^{i,h} = \frac{\frac{\kappa_i^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right)}{E_\omega[\theta_i(\omega)]} + c_i^R}{\frac{\kappa_i^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right)}{E_\omega[\theta_i(\omega)]} + c_i^R + \frac{\kappa_j^{R'}(0)}{E_\omega[\theta_j(\omega)]} + c_j^R}$$

$$\alpha_j^{i,h} = \frac{\frac{\kappa_j^{R'}(0)}{E_\omega[\theta_j(\omega)]} + c_j^R}{\frac{\kappa_i^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right)}{E_\omega[\theta_i(\omega)]} + c_i^R + \frac{\kappa_j^{R'}(0)}{E_\omega[\theta_j(\omega)]} + c_j^R}$$

$$\alpha_h^{i,h} = 0,$$

with $\{i, j, h\} = \{1, 2, 3\}$. Then, $x_i^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$, $x_j^{*R} = x_h^{*R} = 0$, and for vectors with $\alpha_j = \alpha_j^{i,h} + \epsilon$, $\alpha_i = \alpha_i^{i,h} - \epsilon$, $\alpha_h = 0$, $\epsilon > 0$ and small, we have $x_i^{*R}, x_j^{*R} > 0$, $x_h^{*R} = 0$. This follows immediately from Theorem 4.6.

Next, consider vectors $\alpha = [\alpha_1 \quad \alpha_2 \quad \alpha_3]^\top$ with all αs positive. We find for each technology a vector such that this technology is the sole investor while the other two have zero investments. There are many such vectors. However, we are going to consider a very specific one for each technology. We determine a vector such that, when slightly decreasing the α of the investing technology while slightly increasing the α of one of the other technologies, the latter technology will enter the mixture.

The vector $\alpha^i \in \mathbb{R}^3$ is given by

$$\alpha^i_i = \frac{\frac{\kappa_i^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right)}{E_\omega[\theta_i(\omega)]} + c_i^R}{\frac{\kappa_i^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right)}{E_\omega[\theta_i(\omega)]} + c_i^R + \frac{\kappa_j^{R'}(0)}{E_\omega[\theta_j(\omega)]} + c_j^R + \frac{\kappa_h^{R'}(0)}{E_\omega[\theta_h(\omega)]} + c_h^R}$$

$$\alpha^i_j = \frac{\frac{\kappa_j^{R'}(0)}{E_\omega[\theta_j(\omega)]} + c_j^R}{\frac{\kappa_i^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right)}{E_\omega[\theta_i(\omega)]} + c_i^R + \frac{\kappa_j^{R'}(0)}{E_\omega[\theta_j(\omega)]} + c_j^R + \frac{\kappa_h^{R'}(0)}{E_\omega[\theta_h(\omega)]} + c_h^R}$$

$$\alpha^i_h = \frac{\frac{\kappa_h^{R'}(0)}{E_\omega[\theta_h(\omega)]} + c_h^R}{\frac{\kappa_i^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right)}{E_\omega[\theta_i(\omega)]} + c_i^R + \frac{\kappa_j^{R'}(0)}{E_\omega[\theta_j(\omega)]} + c_j^R + \frac{\kappa_h^{R'}(0)}{E_\omega[\theta_h(\omega)]} + c_h^R}$$

with $\{i, j, h\} = \{1, 2, 3\}$. Then, $x_i^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$, $x_j^{*R} = x_h^{*R} = 0$. For vectors with $\alpha_j = \alpha^i_j + \epsilon$, $\alpha_i = \alpha^i_i - \epsilon$, $\alpha_h = \alpha^i_h$, $\epsilon > 0$ and small, we have $x_i^{*R}, x_j^{*R} > 0$, $x_h^{*R} \geq 0$. Note that increasing α_j could result in a positive investment in technology h without increasing α_h. Similarly, for vectors with $\alpha_h = \alpha^i_h + \epsilon$, $\alpha_i = \alpha^i_i - \epsilon$, $\alpha_j = \alpha^i_j$, $\epsilon > 0$ and small, we have $x_i^{*R}, x_h^{*R} > 0$, $x_j^{*R} \geq 0$. These results are formally proven in the next theorem.

Theorem 4.7. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$ and let $K^R = \{1, 2, 3\}$. Assume that in order to get a benchmark value for the feed-in tariffs, the regulator imposes a renewable obligation (4.18) with $\phi \in (0, 1)$ on the first stage, along with the pricing scheme for the fixed price feed-in policy (4.15). Choose $i, j, h \in \{1, 2, 3\}$ such that $\{i, j, h\} = \{1, 2, 3\}$, and let $\bar{\tau}_k^R = \alpha^i_k \sigma^*$, $k \in K^R$. Assume non-linear convex investment cost functions for all three renewable technologies. The following statements hold at an equilibrium:*

(i) $\sigma^* = \tilde{\sigma}^* := \frac{\kappa_i^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right)}{E_\omega[\theta_i(\omega)]} + c_i^R + \frac{\kappa_j^{R'}(0)}{E_\omega[\theta_j(\omega)]} + c_j^R + \frac{\kappa_h^{R'}(0)}{E_\omega[\theta_h(\omega)]} + c_h^R.$

(ii) $x_i^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$, $x_j^{*R} = x_h^{*R} = 0$.

(iii) If we take $\bar{\tau}_i^R = (\alpha^i_i - \epsilon)\sigma^*$ *and* $\bar{\tau}_j^R = (\alpha^i_j + \epsilon)\sigma^*$, $\epsilon > 0$ *and small, then* $x_j^{*R} > 0$.

Proof: (i) Suppose $\sigma^* < \tilde{\sigma}^*$. We know that since $\phi > 0$, at least one renewable technology has a positive investment. Suppose $x_i^{*R} > 0$. Then by Lemma 4.3

$y_i^{*R}(\omega) = \theta_i(\omega)x_i^{*R} > 0$ and by (4.16b) we have $\beta_i^{*R}(\omega) = \alpha_i^i\sigma^* - c_i^R$. This gives

$$E_\omega[\theta_i(\omega)\beta_i^{*R}(\omega)] = E_\omega[\theta_i(\omega)](\alpha_i^i\sigma^* - c_i^R) < E_\omega[\theta_i(\omega)](\alpha_i^i\tilde{\sigma}^* - c_i^R) =$$

$$\kappa_i^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right),$$

meaning that for (4.17b) to be satisfied for $k = i$ we must have $x_i^{*R} < \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$. By Corollary 4.2, we must have that one of the other technologies has a positive investment as well. Suppose $x_j^{*R} > 0$. We have $\beta_j^{*R}(\omega) = \alpha_j^i\sigma^* - c_j^R$ for all $\omega \in \Omega$. This gives

$$E_\omega[\theta_j(\omega)\beta_j^{*R}(\omega)] = E_\omega[\theta_j(\omega)](\alpha_j^i\sigma^* - c_j^R) < E_\omega[\theta_j(\omega)](\alpha_j^i\tilde{\sigma}^* - c_j^R) = \kappa_j^{R'}(0),$$

and hence the left-hand side of (4.17b) is nonbinding and consequently $x_j^{*R} = 0$. Similarly, we have $x_h^{*R} = 0$ yielding a contradiction. Hence, $\sigma^* \geq \tilde{\sigma}^*$. Suppose $\sigma^* > \tilde{\sigma}^*$. By (4.16b) for $k = i$ we have $\beta_i^{*R}(\omega) \geq \alpha_i^i\sigma^* - c_i^R$ for all $\omega \in \Omega$. This gives

$$E_\omega[\theta_i(\omega)\beta_i^{*R}(\omega)] \geq E_\omega[\theta_i(\omega)](\alpha_i^i\sigma^* - c_i^R) > E_\omega[\theta_i(\omega)](\alpha_i^i\tilde{\sigma}^* - c_i^R) =$$

$$\kappa_i^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right),$$

meaning that for (4.17b) to be satisfied for $k = i$ we must have $x_i^{*R} > \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$. This is in contradiction with Corollary 4.2.

(ii) Suppose $x_j^{*R} > 0$. We have

$$E_\omega[\theta_j(\omega)\beta_j^{*R}(\omega)] = E_\omega[\theta_j(\omega)](\alpha_j^i\sigma^* - c_j^R) = E_\omega[\theta_j(\omega)](\alpha_j^i\tilde{\sigma}^* - c_j^R) =$$

$$\kappa_j^{R'}(0) < \kappa_j^{R'}(x_j^{*R}),$$

and hence the left-hand side of (4.17b) is nonbinding for $k = j$ and consequently $x_j^{*R} = 0$. The same holds for technology h, meaning that $x_h^{*R} = 0$ and by Corollary 4.2 that $x_i^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$.

(iii) Changing the αs may result in a different σ^*. We distinguish between two cases, namely $\sigma^* \geq \tilde{\sigma}^*$ and $\sigma^* < \tilde{\sigma}^*$. In case $\sigma^* \geq \tilde{\sigma}^*$, obviously $(\alpha_j^i + \epsilon)\sigma^* > \alpha_j^i\tilde{\sigma}^*$. Suppose $x_j^{*R} = 0$. We have

$$E_\omega[\theta_j(\omega)\beta_j^{*R}(\omega)] \geq E_\omega[\theta_j(\omega)]((\alpha_j^i + \epsilon)\sigma^* - c_j^R) > E_\omega[\theta_j(\omega)](\alpha_j^i\tilde{\sigma}^* - c_j^R) =$$

$$\kappa_j^{R'}(0),$$

and hence the left-hand side of (4.17b) is violated for $k = j$. In case $\sigma^* < \tilde{\sigma}^*$, for technology i we have $(\alpha_i^i - \epsilon)\sigma^* < \alpha_i^i \tilde{\sigma}^*$. This implies

$$E_\omega[\theta_i(\omega)\beta_i^{*R}(\omega)] < \kappa_i^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right),$$

meaning that $x_i^{*R} < \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$ and hence that one of the other technologies must have a positive investment. Since technology h still has the same α_h, technology j should have a positive investment. In fact, $(\alpha_j^i + \epsilon)\sigma^* > \alpha_j^i \tilde{\sigma}^*$ regardless of the lower σ^*. □

We have found nine special vectors for which only one technology will be invested in. We know that when moving in a certain direction in each of them, another technology may enter the mixture. We next use this to derive some generalized results on the optimal technology mixtures for different choices of the parameter vectors. In particular, we characterize the areas in which only one renewable technology is invested in and analyze the line segments in between some of the points.

In order to simplify the notation and the proofs, we define for $i, j \in \{1, 2, 3\}$

$$z_j^i = \begin{cases} \frac{\kappa_i^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right)}{E_\omega[\theta_i(\omega)]} + c_i^R & \text{if} \quad i = j \\ \frac{\kappa_j^{R'}(0)}{E_\omega[\theta_j(\omega)]} + c_j^R & \text{if} \quad i \neq j. \end{cases} \tag{4.38}$$

This allows us to write the vectors $\alpha^{i,j}$ and α^i as follows:

$$\alpha_i^{i,h} = \frac{z_i^i}{z_i^i + z_j^i}$$

$$\alpha_j^{i,h} = \frac{z_j^i}{z_i^i + z_j^i}$$

$$\alpha_h^{i,h} = 0$$

$$\alpha_i^i = \frac{z_i^i}{z_i^i + z_j^i + z_h^i}$$

$$\alpha_j^i = \frac{z_j^i}{z_i^i + z_j^i + z_h^i}$$

$$\alpha_h^i = \frac{z_h^i}{z_i^i + z_j^i + z_h^i}$$

with $\{i, j, h\} = \{1, 2, 3\}$. We next show that if we take any convex combination of the vectors e^i, α^i, $\alpha^{i,j}$, and $\alpha^{i,h}$, with e^i the unit vector with ith element equal to 1, we have $x_i^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$ and $x_j^{*R} = x_h^{*R} = 0$. We denote the set of all convex combinations of e^i, α^i, $\alpha^{i,j}$, and $\alpha^{i,h}$ by $Conv\{e^i, \alpha^i, \alpha^{i,j}, \alpha^{i,h}\}$, the convex hull of the four vectors.

Theorem 4.8. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$ and let $K^R = \{1, 2, 3\}$. Assume that in order to get a benchmark value for the feed-in tariffs, the regulator imposes a renewable obligation (4.18) with $\phi \in (0, 1)$ on the first stage, along with the pricing scheme for the fixed price feed-in policy (4.15). Choose $i, j, h \in \{1, 2, 3\}$ such that $\{i, j, h\} = \{1, 2, 3\}$. Let $\tilde{\alpha} \in Conv\{e^i, \alpha^i, \alpha^{i,j}, \alpha^{i,h}\}$, and let $\bar{\tau}_k^R = \tilde{\alpha}_k \sigma^*$, $k \in K^R$. Assume non-linear convex investment cost functions for all three renewable technologies. Then $x_i^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$ and $x_j^{*R} = x_h^{*R} = 0$.*

Proof: Let $\tilde{\alpha} = \psi_1 e^i + \psi_2 \alpha^i + \psi_3 \alpha^{i,j} + \psi_4 \alpha^{i,h}$ with $\psi_k \geq 0$, $k \in \{1, 2, 3, 4\}$, and $\sum_{k=1}^{4} \psi_k = 1$. We thus have

$$\tilde{\alpha}_i = \psi_1 + \frac{\psi_2 z_i^i}{z_i^i + z_j^i + z_h^i} + \frac{\psi_3 z_i^i}{z_i^i + z_j^i} + \frac{\psi_4 z_i^i}{z_i^i + z_h^i} = \frac{\psi_1(z_i^i+z_j^i+z_h^i)(z_i^i+z_j^i)(z_i^i+z_h^i) + \psi_2 z_i^i(z_i^i+z_j^i)(z_i^i+z_h^i) + \psi_3 z_i^i(z_i^i+z_j^i+z_h^i)(z_i^i+z_h^i) + \psi_4 z_i^i(z_i^i+z_j^i+z_h^i)(z_i^i+z_j^i)}{(z_i^i+z_j^i+z_h^i)(z_i^i+z_j^i)(z_i^i+z_h^i)}, \tag{4.39}$$

$$\tilde{\alpha}_j = \frac{\psi_2 z_j^i}{z_i^i + z_j^i} + \frac{\psi_3 z_j^i}{z_i^i + z_j^i + z_h^i} = \frac{\psi_2 z_j^i (z_i^i + z_j^i) + \psi_3 z_j^i (z_i^i + z_j^i + z_h^i)}{(z_i^i + z_j^i + z_h^i)(z_i^i + z_j^i)}, \tag{4.40}$$

and

$$\tilde{\alpha}_h = \frac{\psi_2 z_h^i}{z_i^i + z_h^i} + \frac{\psi_4 z_h^i}{z_i^i + z_j^i + z_h^i} = \frac{\psi_2 z_h^i (z_i^i + z_h^i) + \psi_4 z_h^i (z_i^i + z_j^i + z_h^i)}{(z_i^i + z_j^i + z_h^i)(z_i^i + z_h^i)}. \tag{4.41}$$

We are going to find an upper bound for σ^* and show that if $x_j^{*R} > 0$ or $x_h^{*R} > 0$ we are not at an equilibrium.

By Corollary 4.2, we know that $x_i^{*R} \leq \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$. Furthermore, at the equilibrium we must have

$$E_\omega[\theta_i(\omega)\beta_i^{*R}(\omega)] \leq \kappa_i^{R'}(x_i^{*R})$$

by (4.17b), and we have $\beta_i^{*R}(\omega) \geq \tilde{\alpha}_i\sigma^* - c_i^R$ by (4.16b). This and the fact that $\kappa_i^{R'}(x)$ is increasing in x yields

$$\kappa_i^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right) \geq \kappa_i^{R'}(x_i^{*R}) \geq E_\omega[\theta_i(\omega)](\tilde{\alpha}_i\sigma^* - c_i^R).$$

We obtain an upper bound for σ^*, namely

$$\sigma^* \leq \frac{z_i^i}{\tilde{\alpha}_i}.$$

Now suppose $x_j^{*R} > 0$. Using the upper bound, (4.39), and (4.40), we obtain

$$\tilde{\alpha}_j\sigma^* \leq \frac{z_i^i\tilde{\alpha}_j}{\tilde{\alpha}_i} =$$

$$\frac{z_i^i(z_i^i+z_h^i)(\psi_2 z_j^i(z_i^i+z_j^i)+\psi_3 z_j^i(z_i^i+z_j^i+z_h^i))}{\psi_1(z_i^i+z_j^i+z_h^i)(z_i^i+z_j^i)(z_i^i+z_h^i)+\psi_2 z_i^i(z_i^i+z_j^i)(z_i^i+z_h^i)+\psi_3 z_i^i(z_i^i+z_j^i+z_h^i)(z_i^i+z_h^i)+\psi_4 z_i^i(z_i^i+z_j^i+z_h^i)(z_i^i+z_j^i)}$$

$$\leq \frac{z_i^i(z_i^i+z_h^i)(\psi_2 z_j^i(z_i^i+z_j^i)+\psi_3 z_j^i(z_i^i+z_j^i+z_h^i))}{\psi_2 z_i^i(z_i^i+z_j^i)(z_i^i+z_h^i)+\psi_3 z_i^i(z_i^i+z_j^i+z_h^i)(z_i^i+z_h^i)} = z_j^i.$$

This yields

$$E_\omega[\theta_j(\omega)\beta_j^{*R}(\omega)] = E_\omega[\theta_j(\omega)](\tilde{\alpha}_j\sigma^* - c_j^R) \leq \kappa_j^{R'}(0) < \kappa_j^{R'}(x_j^{*R}),$$

which means that the left-hand side of (4.17b) is nonbinding for $k = j$, which contradicts $x_j^{*R} > 0$. In the same way, $x_h^{*R} > 0$ leads to a contradiction. By Corollary 4.2, $x_i^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$. $\square$

By Theorem 4.6 we know exactly what happens on the line segments $[e^i, e^j]$, $[e^i, e^h]$, and $[e^j, e^h]$. Next, we consider the line segment $[\alpha^i, \alpha^{i,j}]$ in more detail. This line segment is in the convex hull of $e^i, \alpha^i, \alpha^{i,j}$, and $\alpha^{i,h}$ and therefore we can apply Theorem 4.8. In addition, we show that on one side of this line segment, both technologies i and j will be in the optimal mixture.

Theorem 4.9. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$ and let $K^R = \{1, 2, 3\}$. Assume that in order to get a benchmark value for the feed-in tariffs, the regulator imposes a renewable obligation (4.18) with $\phi \in (0, 1)$ on the first stage, along with the pricing scheme for the fixed* *price feed-in policy (4.15). Choose $i, j, h \in \{1,2,3\}$ such that $\{i, j, h\} = \{1,2,3\}$. Let $\tilde{\alpha} \in int\ Conv\{\alpha^i, \alpha^{i,j}\}$, and let $\tilde{\tau}_k^R = \tilde{\alpha}_k \tilde{\sigma}^*$, $k \in K^R$. Let $\hat{\alpha} = \tilde{\alpha} + \epsilon(e^j - e^i)$ with $\epsilon > 0$ and small, and let $\hat{\tau}_k^R = \hat{\alpha}_k \hat{\sigma}^*$, $k \in K^R$. Assume non-linear convex investment cost functions for all three renewable technologies. Then the following statements hold at an equilibrium:*

(i) If $\alpha = \tilde{\alpha}$, then $\tilde{x}_i^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$, $\tilde{x}_j^{*R} = \tilde{x}_h^{*R} = 0$, $\tilde{\alpha}_i \tilde{\sigma}^* = z_i^i$, $\tilde{\alpha}_j \tilde{\sigma}^* = z_j^i$, and $\tilde{\alpha}_h \tilde{\sigma}^* < z_h^i$.

(ii) If $\alpha = \hat{\alpha}$, then $\hat{x}_i^{*R} < \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$, $\hat{x}_j^{*R} > 0$, and $\hat{x}_h^{*R} = 0$.

Proof: (i) Let $\psi \in (0,1)$ be such that $\tilde{\alpha} = \psi\alpha^i + (1-\psi)\alpha^{i,j}$. Obviously, $\tilde{\alpha} \in Conv\{e^i, \alpha^i, \alpha^{i,j}, \alpha^{i,h}\}$, so by Theorem 4.8 the optimal investment quantities immediately follow. Since $\tilde{x}_i^{*R} > 0$, we have

$$E_\omega[\theta_i(\omega)](\tilde{\alpha}_i \tilde{\sigma}^* - c_i^R) = \kappa_i^{R\prime}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right),$$

which shows that $\tilde{\alpha}_i \tilde{\sigma}^* = z_i^i$. Then we have

$$\tilde{\alpha}_j \tilde{\sigma}^* = \frac{z_i^i \tilde{\alpha}_j}{\tilde{\alpha}_i} =$$

$$z_i^i \left(\frac{\psi z_j^i (z_i^i + z_j^i + z_h^i) + (1-\psi) z_j^i (z_i^i + z_j^i)}{(z_i^i + z_j^i + z_h^i)(z_i^i + z_j^i)}\right) \cdot$$

$$\left(\frac{(z_i^i + z_j^i + z_h^i)(z_i^i + z_j^i)}{\psi z_i^i (z_i^i + z_j^i + z_h^i) + (1-\psi) z_i^i (z_i^i + z_j^i)}\right) = z_j^i.$$

Similarly,

$$\tilde{\alpha}_h \tilde{\sigma}^* = \frac{z_i^i \tilde{\alpha}_h}{\tilde{\alpha}_i} =$$

$$z_i^i \left(\frac{(1-\psi) z_h^i}{(z_i^i + z_j^i + z_h^i)}\right)\left(\frac{(z_i^i + z_j^i + z_h^i)(z_i^i + z_j^i)}{\psi z_i^i (z_i^i + z_j^i + z_h^i) + (1-\psi) z_i^i (z_i^i + z_j^i)}\right) =$$

$$\frac{(1-\psi) z_j^i (z_i^i + z_j^i)}{\psi (z_i^i + z_j^i + z_h^i) + (1-\psi)(z_i^i + z_j^i)} < z_h^i.$$

(ii) We show that we have $\hat{\alpha}_j \hat{\sigma}^* > z_j^i$ and hence $\hat{x}_j^{*R} > 0$. Suppose $\hat{\alpha}_j \hat{\sigma}^* \le z_j^i$. Since $\hat{\alpha}_j > \tilde{\alpha}_j$, this implies $\hat{\sigma}^* < \tilde{\sigma}^*$. Then, using that $\hat{\alpha}_i < \tilde{\alpha}_i$ and $\hat{\alpha}_h = \tilde{\alpha}_h$, we obtain $\hat{\alpha}_i\hat{\sigma}^* < \hat{\alpha}_i\tilde{\sigma}^* < \tilde{\alpha}_i\tilde{\sigma}^* = z_i^i$ and $\hat{\alpha}_h\hat{\sigma}^* < \hat{\alpha}_h\tilde{\sigma}^* = \tilde{\alpha}_h\tilde{\sigma}^* < z_h^i$. Hence we have $\hat{x}_i^{*R} < \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$ and $\hat{x}_h^{*R} = 0$. By Corollary 4.2, $x_j^{*R} = 0$ is not an equilibrium. □

We next consider the line segment $[\alpha^i, \alpha^j]$ and the area $Conv\{\alpha^i, \alpha^j, \alpha^h\}$ for some $\{i, j, h\} = \{1, 2, 3\}$. On the line segment, either only technologies i and j are in the mixture, or all three technologies are in the mixture. The mixture depends on the structure of the cost function. We can show that when we have quadratic investment cost and hence linear marginal investment cost for all technologies, only technologies i and j are in the mixture. Furthermore, when slightly increasing the α_h coefficient, technology h enters the mixture as well.

Theorem 4.10. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$ and let $K^R = \{1, 2, 3\}$. Assume that in order to get a benchmark value for the feed-in tariffs, the regulator imposes a renewable obligation (4.18) with $\phi \in (0, 1)$ on the first stage, along with the pricing scheme for the fixed price feed-in policy (4.15). Choose $i, j, h \in \{1, 2, 3\}$ such that $\{i, j, h\} = \{1, 2, 3\}$. Let $\tilde{\alpha} \in int\ Conv\{\alpha^i, \alpha^j\}$, and let $\tilde{\tau}_k^R = \tilde{\alpha}_k\tilde{\sigma}^*$, $k \in K^R$. Let $\hat{\alpha} = \tilde{\alpha} + \epsilon(2e^h - e^i - e^j)$ with $\epsilon > 0$ and small, and let $\hat{\tau}_k^R = \hat{\alpha}_k\tilde{\sigma}^*$, $k \in K^R$. Assume quadratic investment cost functions for all three renewable technologies where $\kappa_k^{R'}(x) = s_k x + \bar{\kappa}_k^R$, $k \in K^R$, with $\bar{\kappa}_k^R$ and s_k, $k \in K^R$, positive constants. Then the following statements hold at an equilibrium:*

(i) If $\alpha = \tilde{\alpha}$, then $\tilde{x}_i^{*R} > 0$, $\tilde{x}_j^{*R} > 0$, $\tilde{x}_h^{*R} = 0$, and $\tilde{\alpha}_h\tilde{\sigma}^* = z_h^i$.

(ii) If $\alpha = \hat{\alpha}$, then $\hat{x}_i^{*R} > 0$, $\hat{x}_j^{*R} > 0$, and $\hat{x}_h^{*R} > 0$.

Proof: (i) Let $\psi \in (0, 1)$ be such that $\tilde{\alpha} = \psi\alpha^i + (1 - \psi)\alpha^j$. By (4.17b) and (4.16b) we have for all $k \in K^R$

$$s_k\tilde{x}_k^{*R} + \bar{\kappa}_k^R \geq E_\omega[\theta_k(\omega)\tilde{\beta}_k^{*R}(\omega)] \geq E_\omega[\theta_k(\omega)](\tilde{\alpha}_k\tilde{\sigma}^* - c_k^R),$$

where equalities hold whenever $\tilde{x}_k^{*R} > 0$. Since we can write $\bar{\kappa}_k^R = \kappa_k^{R'}(0) = E_\omega[\theta_k(\omega)](z_k^i - c_k^R)$, $k \in K^R$, $k \neq i$, this implies that for all $k \in K^R$, $k \neq i$, we have

$$\tilde{x}_k^{*R} \geq \frac{1}{s_k} E_\omega[\theta_k(\omega)](\tilde{\alpha}_k\tilde{\sigma}^* - z_k^i), \tag{4.42}$$

with equality whenever the right hand side is positive. Suppose $\tilde{x}_h^{*R} > 0$. By (4.42), this implies $\tilde{\alpha}_h\tilde{\sigma}^* > z_h^i$. This yields

$$\tilde{x}_i^{*R} \geq \frac{1}{s_i} E_\omega[\theta_i(\omega)](\tilde{\alpha}_i\tilde{\sigma}^* - z_i^j) > \frac{1}{s_i} E_\omega[\theta_i(\omega)](\frac{\tilde{\alpha}_i z_h^i}{\tilde{\alpha}_h} - z_i^j) =$$

$$\frac{1}{s_i}E_\omega[\theta_i(\omega)]\frac{\psi(z_i^i - z_i^j)(z_i^j + z_j^j + z_h^i)}{\psi(z_i^j + z_j^j + z_h^i) + (1-\psi)(z_i^i + z_j^i + z_h^i)} =$$

$$\frac{1}{s_i}E_\omega[\theta_i(\omega)]\frac{\psi\left(\frac{\kappa_i^{R\prime}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right)+\kappa_i^{R\prime}(0)}{E_\omega[\theta_i(\omega)]} + c_i^R - c_i^R\right)(z_i^j + z_j^j + z_h^i)}{\psi(z_i^j + z_j^j + z_h^i) + (1-\psi)(z_i^i + z_j^i + z_h^i)} =$$

$$\frac{\psi\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}(z_i^j + z_j^j + z_h^i)}{\psi(z_i^j + z_j^j + z_h^i) + (1-\psi)(z_i^i + z_j^i + z_h^i)}.$$

Similarly,

$$\tilde{x}_j^{*R} \geq \frac{1}{s_j}E_\omega[\theta_j(\omega)](\tilde{\alpha}_j\tilde{\sigma}^* - z_j^i) > \frac{(1-\psi)\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_j(\omega)]}(z_i^i + z_j^i + z_h^i)}{\psi(z_i^j + z_j^j + z_h^i) + (1-\psi)(z_i^i + z_j^i + z_h^i)}.$$

Hence, the total expected production becomes

$$E_\omega[\theta_i(\omega)]\tilde{x}_i^{*R} + E_\omega[\theta_j(\omega)]\tilde{x}_j^{*R} + E_\omega[\theta_h(\omega)]\tilde{x}_h^{*R} >$$

$$E_\omega[\theta_i(\omega)]\tilde{x}_i^{*R} + E_\omega[\theta_j(\omega)]\tilde{x}_j^{*R} > \phi E_\omega[d(\omega)].$$

This contradicts Corollary 4.2. Hence, $\tilde{x}_h^{*R} = 0$. Similarly, along the same lines, $\tilde{\alpha}_h\tilde{\sigma}^* < z_h^i$ leads to

$$E_\omega[\theta_i(\omega)]\tilde{x}_i^{*R} + E_\omega[\theta_j(\omega)]\tilde{x}_j^{*R} + E_\omega[\theta_h(\omega)]\tilde{x}_h^{*R} =$$

$$E_\omega[\theta_i(\omega)]\tilde{x}_i^{*R} + E_\omega[\theta_j(\omega)]\tilde{x}_j^{*R} < \phi E_\omega[d(\omega)].$$

We thus have $\tilde{\alpha}_h\tilde{\sigma}^* = z_h^i$. Since the right-hand side in (4.42) is positive for $k = i, j$, $\tilde{x}_i^{*R} > 0$ and $\tilde{x}_j^{*R} > 0$ immediately follow. More specifically,

$$\tilde{x}_i^{*R} = \frac{\psi\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}(z_i^j + z_j^j + z_h^i)}{\psi(z_i^j + z_j^j + z_h^i) + (1-\psi)(z_i^i + z_j^i + z_h^i)} > 0$$

and

$$\tilde{x}_j^{*R} = \frac{(1-\psi)\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_j(\omega)]}(z_i^i + z_j^i + z_h^i)}{\psi(z_i^j + z_j^j + z_h^i) + (1-\psi)(z_i^i + z_j^i + z_h^i)} > 0.$$

(ii) Recall that $0 = \tilde{x}_h^{*R} = \frac{1}{s_h} E_\omega[\theta_h(\omega)](\tilde{\alpha}_h \tilde{\sigma}^* - z_h^i)$. Since $\hat{\alpha}_h > \tilde{\alpha}_h$, either $\hat{\alpha}_h \hat{\sigma}^* - z_h^i > 0$ meaning that $\hat{x}_h^{*R} > 0$ or we have $\hat{\alpha}_h \hat{\sigma}^* - z_h^i = 0$ meaning that $\hat{x}_h^{*R} = 0$ and $\hat{\sigma} < \tilde{\sigma}$. The latter means that $\hat{\tau}_i^R < \tilde{\tau}_i^R$ and $\hat{\tau}_j^R < \tilde{\tau}_j^R$, in other words that the support for technologies i and j decreases. This implies $\hat{x}_i^{*R} < \tilde{x}_i^{*R}$ and $\hat{x}_j^{*R} < \tilde{x}_j^{*R}$, respectively, and hence

$$E_\omega[\theta_i(\omega)]\hat{x}_i^{*R} + E_\omega[\theta_j(\omega)]\hat{x}_j^{*R} < E_\omega[\theta_i(\omega)]\tilde{x}_i^{*R} + E_\omega[\theta_j(\omega)]\tilde{x}_j^{*R} = \phi E_\omega[d(\omega)].$$

This contradicts Corollary 4.2. Furthermore, since $\tilde{x}_i^{*R} > 0$ and $\tilde{x}_j^{*R} > 0$, we can take ϵ small enough such that $\hat{x}_i^{*R} > 0$ and $\hat{x}_j^{*R} > 0$. $\square$

Using Theorems 4.9 and 4.10, we can define the areas in which two and three technologies are in the optimal mixture. This is summarized in the following corollary.

Corollary 4.6. *Consider the problem in Theorem 4.10. The following statements hold at an equilibrium:*

(i) If $\alpha \in$ int Conv$\{\alpha^{i,h}, \alpha^i, \alpha^{j,h}, \alpha^j\} \bigcup$ int Conv$\{\alpha^i, \alpha^j\}$, then $x_i^{*R} > 0$, $x_j^{*R} > 0$, and $x_h^{*R} = 0$.

(ii) If $\alpha \in$ int Conv$\{\alpha^i, \alpha^j, \alpha^h\}$, then $x_i^{*R} > 0$, $x_j^{*R} > 0$, and $x_h^{*R} > 0$.

Proof: (i) Let $\alpha = \psi_1 \alpha^{i,h} + \psi_2 \alpha^i + \psi_3 \alpha^{j,h} + \psi_4 \alpha^j$, with $\sum_{i=1}^{4} \psi_i = 1$, $\psi_1, \psi_3 \geq 0$, and $\psi_2, \psi_4 > 0$. If $\psi_1 = \psi_3 = 0$ then we are on the line segment *int Conv*$\{\alpha^i, \alpha^j\}$ and by (i) in Theorem 4.10, $x_i^{*R} > 0$, $x_j^{*R} > 0$, and $x_h^{*R} = 0$. If not, then $\alpha_h = \psi_2 \alpha_h^i + \psi_4 \alpha_h^j < \psi \alpha_h^i + (1-\psi)\alpha_h^j$ for some $\psi \in (0,1)$. We thus have that α_h is lower than in the case of $\psi_1 = \psi_3 = 0$. Along the same lines as the proof of (i) in Theorem 4.10, $x_h^{*R} = 0$ follows. In addition, by (ii) in Theorem 4.9, $x_i^{*R} > 0$ and $x_j^{*R} > 0$.

(ii) This immediately follows from applying (i) and (ii) in Theorem 4.10 to i and h and to j and h. $\square$

Theorem 4.10 and Corollary 4.6 show that when all cost functions are quadratic, we can analytically characterize the areas for which one, two, and three renewable technologies are investing. In the next section, we give a numerical example. When cost-functions are of a different form, using Theorems 4.7, 4.8, and 4.9 one can determine the feed-in parameters for which one technology is singled out, as well as ranges of feed-in parameters or line segments such that on one side of such line segments a second technology enters the optimal mixture. However, the results from Theorem 4.10 no longer hold, meaning that analytically the feed-in parameters for which any two or three renewable technologies are in the optimal mixture cannot be traced. Our numerical tools provide a way to overcome this, as we show in Section 4.6.4.

For a market with two or three renewable technologies with general non-linear convex investment cost functions, we have determined for which choices of the feed-in parameters a single renewable technology is invested in. We next show that the same results hold when considering n renewable technologies. To simplify notation, we define z_j^i for all $i, j \in K^R = \{1, ..., n\}$ according to (4.38). We define vectors $\alpha^{i,H}$, where $H \subseteq K^R \setminus \{i\}$ is the set of renewable technologies that we give zero support; this set is allowed to be empty. We write $\alpha^i := \alpha^{i,\emptyset}$. We define

$$\alpha_j^{i,H} = \begin{cases} \dfrac{z_j^i}{\sum\limits_{h \notin H} z_h^i} & j \notin H \\ 0 & j \in H. \end{cases}$$

We next state the generalizations of Theorems 4.8 and 4.9. Detailed proofs are omitted as they are similar to the proofs of Theorems 4.8 and 4.9.

Theorem 4.11. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$ and let $K^R = \{1, ..., n\}$. Assume that in order to get a benchmark value for the feed-in tariffs, the regulator imposes a renewable obligation (4.18) with $\phi \in (0, 1)$ on the first stage, along with the pricing scheme for the fixed price feed-in policy (4.15). Let $i \in \{1, ..., n\}$ be given, let*

$$\tilde{\alpha} \in Conv \left\{ \alpha^{i,H} | H \subseteq K^R \setminus \{i\} \right\},$$

and let $\bar{\tau}_k^R = \tilde{\alpha}_k \sigma^*$, $k \in K^R$. Assume non-linear convex investment cost functions for all renewable technologies. Then $x_i^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$ and $x_j^{*R} = 0\ \forall j \in K^R \setminus \{i\}$.

Theorem 4.12. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$ and let $K^R = \{1, ..., n\}$. Assume that in order to get a benchmark value for the feed-in tariffs, the regulator imposes a renewable obligation*

(4.18) with $\phi \in (0,1)$ on the first stage, along with the pricing scheme for the fixed price feed-in policy (4.15). For some $i \in K^R$, let $H_j = K^R \setminus \{i,j\}$ for some $j \neq i$, let $\tilde{\alpha} \in int\ Conv\{\alpha^i, \alpha^{i,H_j}\}$, and let $\tilde{\tau}_k^R = \tilde{\alpha}_k \tilde{\sigma}^*$, $k \in K^R$. Let $\hat{\alpha} = \tilde{\alpha} + \epsilon(e^j - e^i)$ with $\epsilon > 0$ and small, and let $\hat{\tau}_k^R = \hat{\alpha}_k \hat{\sigma}^*$, $k \in K^R$. Assume non-linear convex investment cost functions for all renewable technologies. Then the following statements hold at an equilibrium:

(i) If $\alpha = \tilde{\alpha}$, then $\tilde{x}_i^{*R} = \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$, $\tilde{x}_k^{*R} = 0$ for $k \neq i$, $\tilde{\alpha}_i \tilde{\sigma}^* = z_i^i$, $\tilde{\alpha}_j \tilde{\sigma}^* = z_j^i$, and $\tilde{\alpha}_h \tilde{\sigma}^* < z_h^i$, $h \in H_j$.

(ii) If $\alpha = \hat{\alpha}$, then $0 < \hat{x}_i^{*R} < \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}$, $\hat{x}_j^{*R} > 0$, and $\hat{x}_h^{*R} = 0$, $h \in H_j$.

4.6 Numerical Study

In this section we introduce a numerical framework to show how the theory from Section 4.5 can be applied in practice. We deal with both linear and non-linear convex investment cost functions and provide numerical evidence for our conclusion that with linear investment cost functions a system of feed-in tariffs cannot guarantee that certain targets on renewable electricity will be met. Furthermore, for the case of quadratic cost functions, we show which technologies are in the optimal investment mixture for any choice of the feed-in parameters. Recall that for non-linear non-quadratic investment cost functions, analytically we can only determine the parameter choices for which one technology is singled out. Numerically however, we do obtain information on the optimal technology mixture for any parameter choice by doing numerical experiments. We finally use our observations in this numerical study to make a theoretical statement.

We consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage. We impose renewable obligation (4.18) with $\phi \in (0,1)$ on the first stage, along with the pricing scheme for the fixed price feed-in policy (4.15), as to get a benchmark value for the feed-in tariffs. We show the effects of having different choices of the parameter vector α on technology choices in case of linear, quadratic, and two other non-linear convex investment cost functions, and (partly) characterize the equilibria. In order to deal with the expectations in the model, we use a sampling technique. We generate 3000 random samples for both the uncertain demand and the uncertain availability of renewable resources, and replace expectations by sample averages. We solve the resulting problem using the PATH solver, see Ferris and Munson

(2000). Computation times are approximately 20 minutes for each problem we solve on a 300MHz Pentium-II with 1 GB RAM.

4.6.1 Experimental Data

We assume that there are five firms, each investing in a unique technology. Renewable technologies onshore wind (ONW), offshore wind (OFFW), and solar (SOL) are used by firms 1, 2, and 3, respectively. Non-renewable technologies coal and closed cycle gas turbine (CCGT) are used by firms 4 and 5, respectively. There are no network limitations and no limits on the maximum investment in each technology. Table 4.1 contains the characteristics of these technologies, consisting of per unit production costs (c_k), fixed investment costs for non-renewable technologies (κ_k^N, $k = 4, 5$), parameters of the probability distributions of available renewable capacities, and its sample averages. The functions $F_k(x, \omega) = \theta_k(\omega)x$ have $\theta_k(\omega)$ sampled from a uniform distribution with lower bound a_k and upper bound b_k, $k = 1, 2, 3$. By $\bar{\theta}_k$, $k = 1, 2, 3$, we denote the sample averages.

Table 4.1: Characteristics of the technologies.

	c_k	κ_k^N	a_k	b_k	$\bar{\theta}_k$
ONW	0		.17	.29	0.2303
OFFW	0		.34	.44	0.3902
SOL	0		.05	.17	0.1105
Coal	17.77	27.23			
CCGT	41.28	13.72			

The cost figures are in Euros and are based on data by the European Commission (See European Commission (2008)). The investment cost functions for renewable technologies are specified below. We generated the random availabilities of onshore and offshore wind using the same sample, meaning that onshore and offshore wind realizations are fully correlated. Realizations for demand and solar however, are taken from independent samples. The uniform distributions are chosen out of convenience, and one could choose alternatives that may fit empirical data closely or even use empirical data itself.

Demand is assumed to be uniformly distributed with lower bound 30 and upper bound 35; the sample average demand is 32.4919. For the renewable obligation, we take $\phi = 0.15$, and we choose $VOLL = 10.000$.

4.6.2 Linear Cost Functions

We assume the three renewable technologies to have a linear investment cost function. We take $\kappa_k^R(x) = \bar{\kappa}_k^R x$, $x \geq 0$, $k = 1,2,3$, with $\bar{\kappa}_k^R$ as given in Table 4.2.

Table 4.2: Parameters linear cost functions.

	$\bar{\kappa}_k^R$
ONW	21.28
OFFW	43.88
SOL	77

It can be seen that the data satisfies (4.20) and (4.22). We apply Corollary 4.4 to subsets $H = \{1,2,3\}$, $H = \{1,2\}$, $H = \{1,3\}$, and $H = \{2,3\}$, and adopt the notation we used in case of non-linear convex cost functions. We find

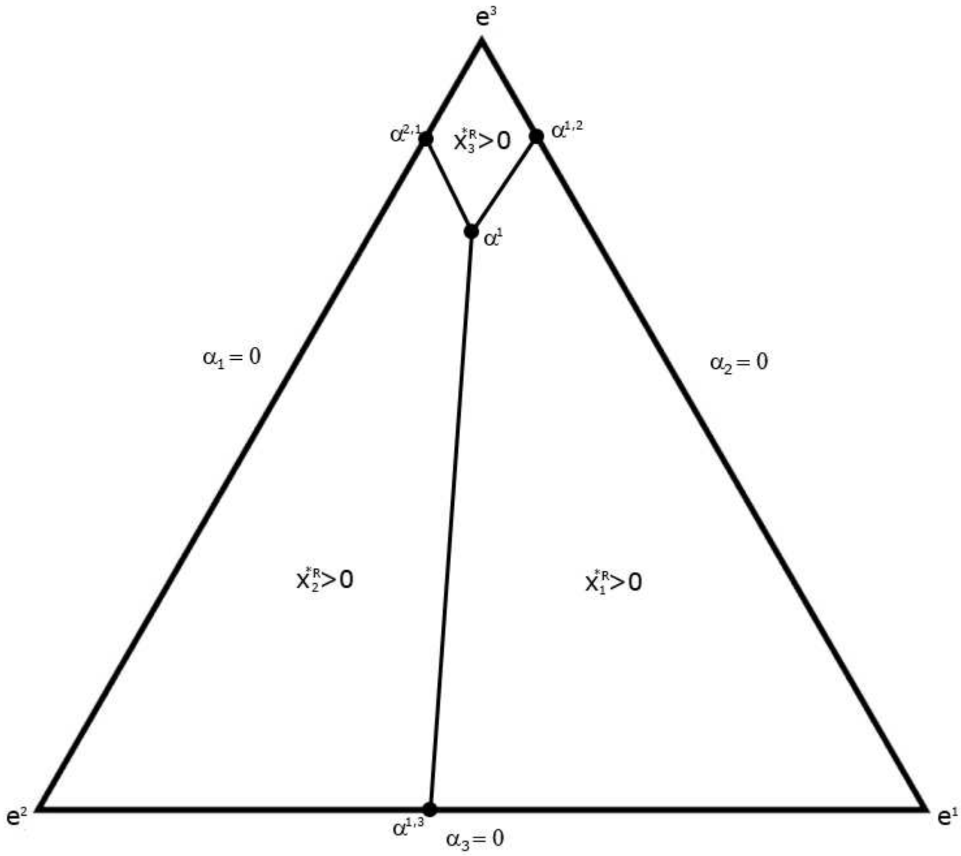
$$\alpha^1 = \alpha^2 = \alpha^3 = \begin{bmatrix} 0.1025 \\ 0.1247 \\ 0.7728 \end{bmatrix}, \quad \alpha^{1,3} = \alpha^{2,3} = \begin{bmatrix} 0.4511 \\ 0.5489 \\ 0 \end{bmatrix},$$

$$\alpha^{1,2} = \alpha^{3,2} = \begin{bmatrix} 0.1171 \\ 0 \\ 0.8829 \end{bmatrix}, \quad \alpha^{2,1} = \alpha^{3,1} = \begin{bmatrix} 0 \\ 0.1390 \\ 0.8610 \end{bmatrix}.$$

Note that (4.28) applied to subsets $H = \{1\}$, $H = \{2\}$, and $H = \{3\}$ leads to the unit vectors e^1, e^2, and e^3, respectively. We depict all these vectors in Figure 4.2 and draw the areas in which each technology is the single technology invested in. The lines within the triangle can be interpreted as follows. On, for example, the line segment $[\alpha^{1,3}, \alpha^1]$, which is in fact also the line segment $[\alpha^{2,3}, \alpha^2]$, both technology 1 and technology 2 can be in the optimal mixture. In fact, there are multiple equilibria on this line segment.

Similarly, we can interpret the point α^1 as the parameter choice for which all three technologies can be in the optimal mixture. While in case of nonlinear investment cost functions there is usually an area for which all three can be in the optimal mixture as we will see later, in case of linear investment cost there is only a single point. Furthermore, in this single point it is not even guaranteed that all three technologies are in the optimal mixture. In particular, if we hand out subsidies equal to $\bar{\tau}_k = \alpha_k^1 \sigma^*$, $k = 1,2,3$, we obtain the following

Figure 4.2: Optimal technology mixtures in case of linear investment cost functions.



solution:

$$x^{*N} = \begin{bmatrix} 27.230 \\ 5.331 \end{bmatrix}, \quad x^{*R} = \begin{bmatrix} 0 \\ 0 \\ 44.103 \end{bmatrix}, \quad \sigma^* = 901.617.$$

We observe that one technology, namely solar, is singled out. A slight deviation from α^1 can lead to another technology being singled out. Finally, if we omit the explicit obligation and fix subsidies at $\bar{\tau}_k = 901.617\alpha_k^1$, $k = 1,2,3$, the problem becomes unbounded since infinite investment in solar is optimal. This would be very costly in reality. On the other hand, if we slightly modify the subsidies to $\bar{\tau}_k = 901.616\alpha_k^1$, $k = 1,2,3$, the problem becomes bounded and feasible, but there is no investment in renewable technologies anymore. Basically, without an explicit obligation there is no guarantee that targets are met. Either we overshoot the target, or there is no investment in renewable technologies at all. This result confirms what we found in Corollary 4.3 and shows how sensitive the problem is to the parameter choices in case of linear cost functions.

4.6.3 Quadratic Cost Functions

Next, we assume the three renewable technologies to have a quadratic investment cost function. We take $\kappa_k^R(x) = 0.5x^2 + \bar{\kappa}_k^R x$, $x \geq 0$, $k = 1,2,3$, with $\bar{\kappa}_k^R$ as given in Table 4.3.

Table 4.3: Parameters quadratic cost functions.

	$\bar{\kappa}_k^R$
ONW	15.28
OFFW	38.88
SOL	65

It can be seen that the parameters satisfy (4.20). We apply the theory from Section 4.5.1 and obtain

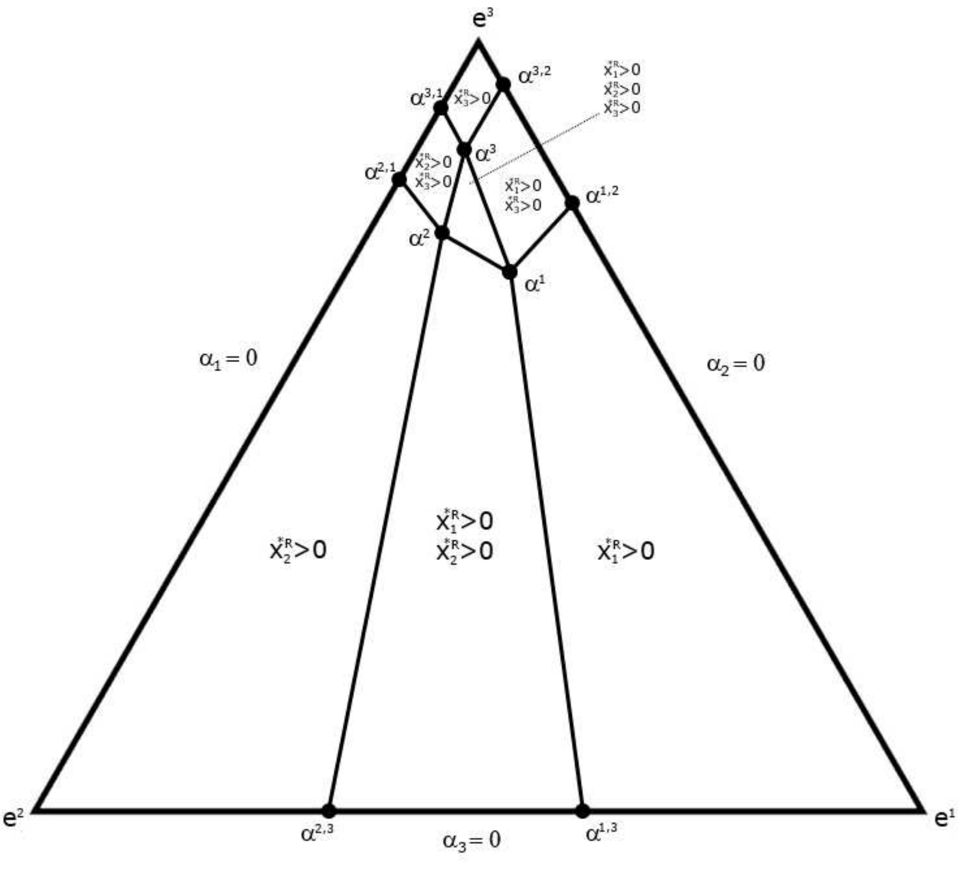
$$\alpha^1 = \begin{bmatrix} 0.1871 \\ 0.1178 \\ 0.6952 \end{bmatrix}, \quad \alpha^2 = \begin{bmatrix} 0.0844 \\ 0.1674 \\ 0.7481 \end{bmatrix}, \quad \alpha^3 = \begin{bmatrix} 0.0575 \\ 0.0864 \\ 0.8561 \end{bmatrix},$$

$$\alpha^{1,3} = \begin{bmatrix} 0.6137 \\ 0.3863 \\ 0 \end{bmatrix}, \quad \alpha^{2,3} = \begin{bmatrix} 0.3351 \\ 0.6649 \\ 0 \end{bmatrix}, \quad \alpha^{1,2} = \begin{bmatrix} 0.2120 \\ 0 \\ 0.7880 \end{bmatrix},$$

$$\alpha^{3,2} = \begin{bmatrix} 0.0630 \\ 0 \\ 0.9370 \end{bmatrix}, \quad \alpha^{2,1} = \begin{bmatrix} 0 \\ 0.1829 \\ 0.8171 \end{bmatrix}, \quad \alpha^{3,1} = \begin{bmatrix} 0 \\ 0.0917 \\ 0.9083 \end{bmatrix}.$$

Using Theorem 4.8 and Corollary 4.6, we determine all the points for which a certain technology mixture is optimal. This is depicted in Figure 4.3. In each point, we find a unique equilibrium solution. For example, on the line segment $Conv\{\alpha^1, \alpha^{1,3}\}$, only technology ONW is in the optimal mixture. On the left of this line segment OFFW is also in the mixture. On the interior of the line segment $Conv\{\alpha^1, \alpha^2\}$ both ONW and OFFW are in the optimal mixture. Above this line segment solar is also in the mixture.

Figure 4.3: Optimal technology mixtures in case of quadratic investment cost functions.



Finally note that when we omit the explicit obligation, set the feed-in tariffs exogenously based on the benchmark values we find, and using an appropriate vector α, the original obligation target is still met as opposed to what we found in case of linear investment cost functions. For example, choosing $\tilde{\alpha} = [0.1 \quad 0.15 \quad 0.75]^\top$, which is in the interior of $Conv\{\alpha^1, \alpha^2, \alpha^3\}$, we obtain

$$x^{*N} = \begin{bmatrix} 27.273 \\ 3.486 \end{bmatrix}, \quad x^{*R} = \begin{bmatrix} 3.7 \\ 9.367 \\ 3.318 \end{bmatrix}, \quad \sigma^* = 824.274.$$

When we omit the obligation and set $\bar{\tau}_k^R = 824.274\tilde{\alpha}_k$ for $k = 1, 2, 3$ as the given feed-in tariffs, we obtain exactly the same solution.

4.6.4 Other Non-linear Convex Cost Functions

We assume the three renewable technologies to have non-quadratic convex investment cost function. First, we consider the cubic functions $\kappa_k^R(x) = 0.01x^3 + \bar{\kappa}_k^R x$, $x \geq 0$, $k = 1, 2, 3$, with $\bar{\kappa}_k^R$ as given in Table 4.3. We find

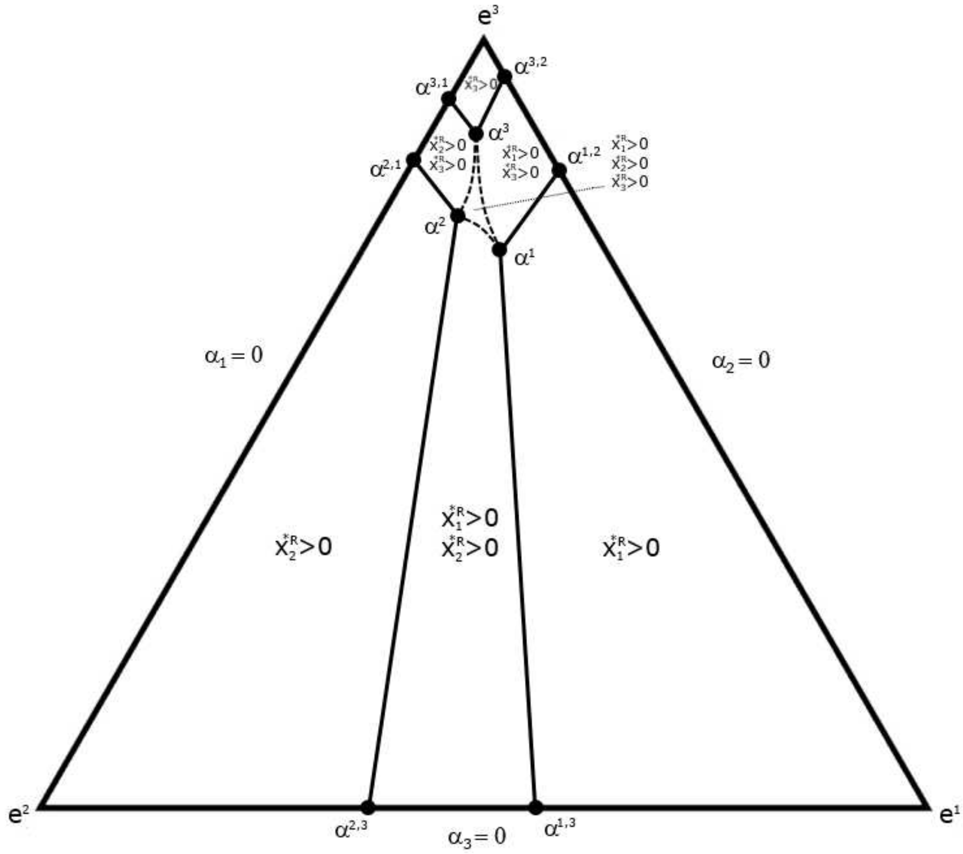
$$\alpha^1 = \begin{bmatrix} 0.1535 \\ 0.1226 \\ 0.7239 \end{bmatrix}, \quad \alpha^2 = \begin{bmatrix} 0.0866 \\ 0.1457 \\ 0.7677 \end{bmatrix}, \quad \alpha^3 = \begin{bmatrix} 0.0518 \\ 0.0777 \\ 0.8705 \end{bmatrix},$$

$$\alpha^{1,3} = \begin{bmatrix} 0.5559 \\ 0.4441 \\ 0 \end{bmatrix}, \quad \alpha^{2,3} = \begin{bmatrix} 0.3728 \\ 0.6272 \\ 0 \end{bmatrix}, \quad \alpha^{1,2} = \begin{bmatrix} 0.1750 \\ 0 \\ 0.8250 \end{bmatrix},$$

$$\alpha^{3,2} = \begin{bmatrix} 0.0561 \\ 0 \\ 0.9439 \end{bmatrix}, \quad \alpha^{2,1} = \begin{bmatrix} 0 \\ 0.1595 \\ 0.8405 \end{bmatrix}, \quad \alpha^{3,1} = \begin{bmatrix} 0 \\ 0.0819 \\ 0.9181 \end{bmatrix}.$$

We can again find the areas for which a single renewable technology is in the mixture, using Theorem 4.8. This is depicted in Figure 4.4. In addition, we know by Theorem 4.9 that on one side of each of these line segments another technology enters the mixture. We cannot, however, use our theorems to determine the exact areas in which two or three technologies are in the optimal mixture. It can be checked that, for example, slightly above $Conv\{\alpha^1, \alpha^2\}$ we still have $x_3^{*R} = 0$. The result from Theorem 4.10 (ii) does not hold here since we have non-quadratic cost functions. The boundary of the non-convex area in which all three technologies are in the optimal mixture is indicated with the dashed curves in Figure 4.4. These curves are obtained by performing a series of numerical experiments for a range of parameters vectors on and close to the line segments $Conv\{\alpha^1, \alpha^2\}$, $Conv\{\alpha^1, \alpha^3\}$, and $Conv\{\alpha^2, \alpha^3\}$.

Figure 4.4: Optimal technology mixtures in case of cubic investment cost functions.



Finally, we consider the functions $\kappa_k^R(x) = 2x^{\frac{3}{2}} + \bar{\kappa}_k^R x$, $x \geq 0$, $k = 1, 2, 3$, with $\bar{\kappa}_k^R$ as given in Table 4.3. We find

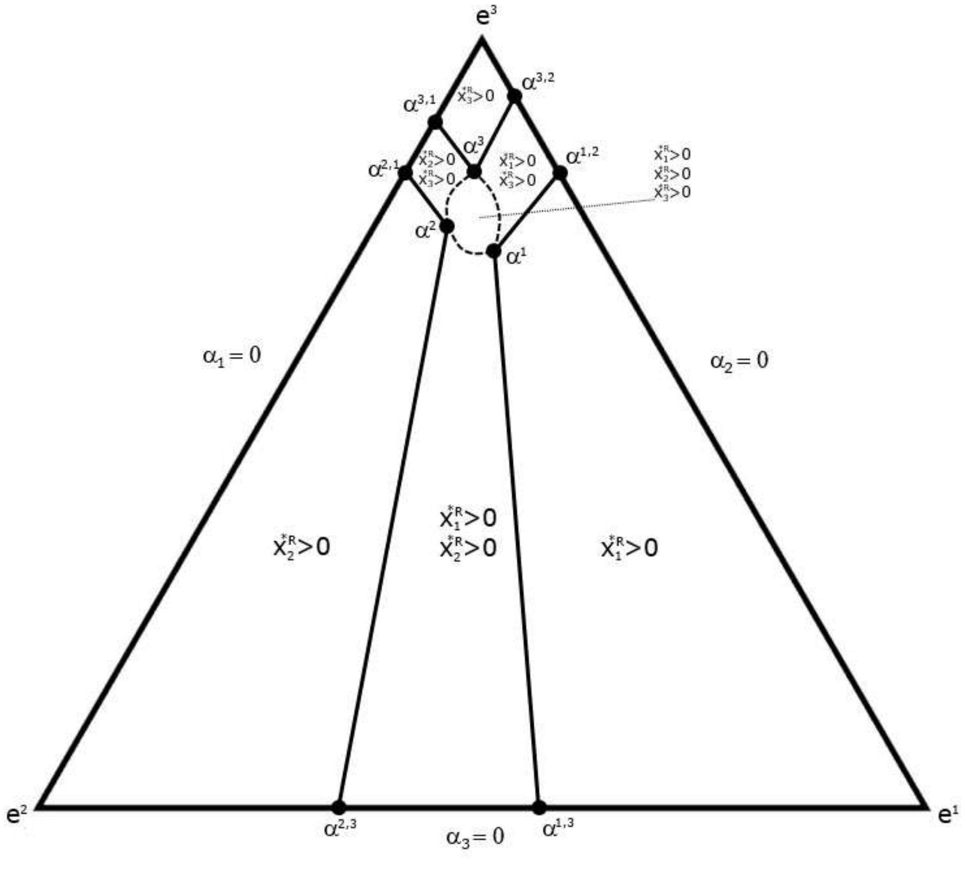
$$\alpha^1 = \begin{bmatrix} 0.1551 \\ 0.1224 \\ 0.7225 \end{bmatrix}, \quad \alpha^2 = \begin{bmatrix} 0.0849 \\ 0.1623 \\ 0.7528 \end{bmatrix}, \quad \alpha^3 = \begin{bmatrix} 0.0710 \\ 0.1066 \\ 0.8224 \end{bmatrix},$$

$$\alpha^{1,3} = \begin{bmatrix} 0.5590 \\ 0.4410 \\ 0 \end{bmatrix}, \quad \alpha^{2,3} = \begin{bmatrix} 0.3435 \\ 0.6565 \\ 0 \end{bmatrix}, \quad \alpha^{1,2} = \begin{bmatrix} 0.1768 \\ 0 \\ 0.8232 \end{bmatrix},$$

$$\alpha^{3,2} = \begin{bmatrix} 0.0795 \\ 0 \\ 0.9205 \end{bmatrix}, \quad \alpha^{2,1} = \begin{bmatrix} 0 \\ 0.1774 \\ 0.8226 \end{bmatrix}, \quad \alpha^{3,1} = \begin{bmatrix} 0 \\ 0.1148 \\ 0.8852 \end{bmatrix}.$$

Using Theorem 4.8, we determine the areas for which a single renewable technology is invested in. Again, we cannot apply Theorem 4.10 since our functions are not quadratic. It can be checked that, for example, on the interior of the line segment $Conv\{\alpha^1, \alpha^2\}$ we have $x_3^{*R} > 0$. The areas where two or three renewable technologies are in the mixture are separated by the dashed curves, see Figure 4.5. Again, the curves are obtained using numerical experiments.

Figure 4.5: Optimal technology mixtures in case of investment cost functions of the form $\kappa_k^R(x) = 2x^{\frac{3}{2}} + \bar{\kappa}_k^R x$, $k = 1, 2, 3$.



Recall that, analytically, we can only partly characterize the equilibrium in case we have non-quadratic investment cost functions; that is, we can fully determine the areas in which only one renewable technology is in the optimal mixture and find line-segments such that on one side of a line segment a second renewable technology enters. However, we cannot apply our theorems to exactly determine the areas in which two or three renewable technologies are in the optimal mixture. Using our numerical tools, we have seen that we can keep track of the optimal mixtures for any choice of the parameter vectors, regardless of the cost functions. These numerical tools thus supplement the theoretical results in this chapter. Moreover, we can make a theoretical generalization based on our observations. In particular, we find a class of investment cost functions for which we obtain a non-convex area similar to the one in Figure 4.4 and a class of investment cost functions for which we obtain a convex area similar to the one in Figure 4.5. We conclude by stating two definitions and the theoretical result, and refer to the appendix for a formal proof.

Definition 4.1. A function $f(x)$ is *superadditive* if for any a, b in its domain we have

$$f(a) + f(b) \leq f(a+b).$$

Definition 4.2. A function $f(x)$ is *subadditive* if for any a, b in its domain we have

$$f(a) + f(b) \geq f(a+b).$$

Theorem 4.13. *Consider the MCP consisting of (4.17) at the first stage and (4.16) at the second stage for every $\omega \in \Omega$ and let $K^R = \{1,2,3\}$. Assume that in order to get a benchmark value for the feed-in tariffs, the regulator imposes a renewable obligation (4.18) with $\phi \in (0,1)$ on the first stage, along with the pricing scheme for the fixed price feed-in policy (4.15). Choose $i, j, h \in \{1,2,3\}$ such that $\{i,j,h\} = \{1,2,3\}$. Let $\alpha \in$ int Conv$\{\alpha^i, \alpha^j\}$, and let $\bar{\tau}_k^R = \alpha_k \sigma^*$, $k \in K^R$. Assume non-linear non-quadratic convex investment cost functions for all three renewable technologies and assume that the non-constant part of each cost function is the same and invertible. In particular, assume that we can write $\kappa_k^{R'}(x) = \tilde{\kappa}(x) + \bar{\kappa}_k^R$, $k \in K^R$, with $\tilde{\kappa}(x)$ an invertible non-decreasing function in x with $\tilde{\kappa}(0) = 0$ and $\bar{\kappa}_k^R$, $k \in K^R$, positive constants. Then the following statements hold at an equilibrium:*

(i) If $\tilde{\kappa}(x)$ superadditive and $\tilde{\kappa}(1) \leq 1$, then $x_h^{*R} = 0$.

(ii) If $\tilde{\kappa}(x)$ subadditive and $\tilde{\kappa}(1) \geq 1$, then $x_h^{*R} > 0$.

Proof: See appendix.

Finally note that the cubic functions $\kappa_k^R(x) = 0.01x^3 + \bar{\kappa}_k^R x$, $k \in K^R$, satisfy the conditions in (i). For each $\{i, j, h\} = \{1, 2, 3\}$ we indeed have $x_h^{*R} = 0$ on the interior of the line segment $[\alpha^i, \alpha^j]$, see Figure 4.4. The functions $\kappa_k^R(x) = 2x^{\frac{3}{2}} + \bar{\kappa}_k^R x$, $k \in K^R$, satisfy the conditions in (ii), and for each $\{i, j, h\} = \{1, 2, 3\}$ we have a positive investment in technology h on the interior of the line segment $[\alpha^i, \alpha^j]$ as shown in Figure 4.5.

4.7 Conclusions

This chapter addressed two instruments used for giving financial incentives for promoting renewable electricity investments, namely renewable certificates and fixed feed-in tariffs. A theoretical drawback of feed-in tariffs is its inability to satisfy certain targets on renewable electricity production in the absence of an explicit obligation on the market.

We presented a mathematical framework and showed that in one particular case, that is when investment cost functions of renewable technologies are linear, indeed a system of (fixed) feed-in tariffs only cannot guarantee that a quota is met and either overshoots the target or does not meet the requirement. Typically either one renewable technology receives more financial support than its levelised cost, resulting in windfall profits and hence overinvestment, or none of the renewable technologies receives sufficient financial support.

We then moved to non-linear convex investment cost functions and showed that under the new assumptions renewable quota can be achieved using a feed-in tariff without explicitly imposing an obligation on the market. Furthermore, the mixture of renewable technologies satisfying the quota can be determined through tariff differentiation by the regulator. We established a number of theoretical results showing for which choices of feed-in tariffs which technologies are in the optimal technology mixture. In case of more than two renewable technologies and when cost functions are non-linear convex and non-quadratic, our numerical tools proved capable of keeping track of the optimal technology mixture corresponding to each choice of feed-in tariffs. We thus provided both theoretical results and numerical tools helping an environmental regulator in finding feed-in tariffs that, as long as cost functions are non-linear convex, guarantee that a target on renewable electricity production can be satisfied without explicitly imposing it on the market and that, for up to three renewable technologies, can achieve any desired optimal renewable technology mixture.

To an extent our results are applicable to other feed-in policies like the premium price policy, the Dutch spot-market gap policy, and the Spanish cap and floor policy. We leave this for further research.

4.8 Appendix: Proof of Theorem 4.13

First recall that we have $\bar{\kappa}_k = \kappa_k^{R'}(0) = E_\omega[\theta_k(\omega)](z_k^i - c_k^R)$, $k \in K^R$, $k \neq i$, and note that

$$z_i^i - z_i^j = \frac{\kappa_k^{R'}\left(\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\right) - \kappa_k^{R'}(0)}{E_\omega[\theta_i(\omega)]} = \frac{\tilde{\kappa}\left(\phi E_\omega[d(\omega)]\right)}{E_\omega[\theta_i(\omega)]}.$$

Using these two statements, we prove (i) and (ii).
(i) Since $\tilde{\kappa}(x)$ is superadditive and invertible, its inverse $\tilde{\kappa}^{-1}(x)$ is subadditive. Suppose $x_h^{*R} > 0$. This implies $\sigma^* > z_h^i/\alpha_h$. Then using (4.17b) and (4.16b) we obtain

$$x_i^{*R} \geq \tilde{\kappa}^{-1}\left(E_\omega[\theta_i(\omega)](\alpha_i\sigma^* - z_i^j)\right) > \tilde{\kappa}^{-1}\left(E_\omega[\theta_i(\omega)](\frac{\alpha_i z_h^i}{\alpha_h} - z_i^j)\right) =$$

$$\tilde{\kappa}^{-1}\left(E_\omega[\theta_i(\omega)]\frac{\psi(z_i^i - z_i^j)(z_i^j + z_j^j + z_h^i)}{\psi(z_i^j + z_j^j + z_h^i) + (1-\psi)(z_i^i + z_j^i + z_h^i)}\right) =$$

$$\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\tilde{\kappa}^{-1}\left(\frac{\psi(z_i^j + z_j^j + z_h^i)}{\psi(z_i^j + z_j^j + z_h^i) + (1-\psi)(z_i^i + z_j^i + z_h^i)}\right).$$

Similarly,

$$x_j^{*R} > \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_j(\omega)]}\tilde{\kappa}^{-1}\left(\frac{(1-\psi)(z_i^i + z_j^i + z_h^i)}{\psi(z_i^j + z_j^j + z_h^i) + (1-\psi)(z_i^i + z_j^i + z_h^i)}\right).$$

Using the subadditivity of $\tilde{\kappa}^{-1}$ and the fact that $\tilde{\kappa}(1) \leq 1$ and hence $\tilde{\kappa}^{-1}(1) \geq 1$, we obtain

$$E_\omega[\theta_i(\omega)]x_i^{*R} + E_\omega[\theta_j(\omega)]x_j^{*R} + E_\omega[\theta_h(\omega)]x_h^{*R} >$$

$$E_\omega[\theta_i(\omega)]x_i^{*R} + E_\omega[\theta_j(\omega)]x_j^{*R} > \phi E_\omega[d(\omega)]\tilde{\kappa}^{-1}(1) \geq \phi E_\omega[d(\omega)].$$

This contradicts Corollary 4.2. Hence, $x_h^{*R} = 0$.
(ii) Since $\tilde{\kappa}(x)$ is subadditive and invertible, its inverse $\tilde{\kappa}^{-1}(x)$ is superadditive. Suppose $x_h^{*R} = 0$. This implies $\sigma^* \leq z_h^i/\alpha_h$. Furthermore, by Theorems 4.7 and 4.8 we know that more than one technology has a positive investment quantity and hence $x_i^{*R} > 0$ and $x_j^{*R} > 0$. Then using (4.17b) and (4.16b) we

obtain

$$x_i^{*R} = \tilde{\kappa}^{-1}\left(E_\omega[\theta_i(\omega)](\alpha_i\sigma^* - z_i^j)\right) \leq \tilde{\kappa}^{-1}\left(E_\omega[\theta_i(\omega)](\frac{\alpha_i z_h^i}{\alpha_h} - z_i^j)\right) =$$

$$\tilde{\kappa}^{-1}\left(E_\omega[\theta_i(\omega)]\frac{\psi(z_i^i - z_i^j)(z_i^j + z_j^j + z_h^i)}{\psi(z_i^j + z_j^j + z_h^i) + (1-\psi)(z_i^i + z_j^i + z_h^i)}\right) =$$

$$\frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_i(\omega)]}\tilde{\kappa}^{-1}\left(\frac{\psi(z_i^j + z_j^j + z_h^i)}{\psi(z_i^j + z_j^j + z_h^i) + (1-\psi)(z_i^i + z_j^i + z_h^i)}\right).$$

Similarly,

$$x_j^{*R} \leq \frac{\phi E_\omega[d(\omega)]}{E_\omega[\theta_j(\omega)]}\tilde{\kappa}^{-1}\left(\frac{(1-\psi)(z_i^i + z_j^i + z_h^i)}{\psi(z_i^j + z_j^j + z_h^i) + (1-\psi)(z_i^i + z_j^i + z_h^i)}\right).$$

Using the superadditivity of $\tilde{\kappa}^{-1}$ and the fact that $\tilde{\kappa}(1) \geq 1$, implying $\tilde{\kappa}^{-1}(1) \leq 1$, we obtain

$$E_\omega[\theta_i(\omega)]x_i^{*R} + E_\omega[\theta_j(\omega)]x_j^{*R} + E_\omega[\theta_h(\omega)]x_h^{*R} <$$

$$\phi E_\omega[d(\omega)]\tilde{\kappa}^{-1}(1) \leq \phi E_\omega[d(\omega)].$$

The strict inequality follows from the superadditivity of $\tilde{\kappa}^{-1}$ and the fact that both evaluation points are nonzero. We have a contradiction with Corollary 4.2. Hence, $x_h^{*R} > 0$. □

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